Photo credit: Hannah Isaac

THE UNIVERSITY OF ARIZONA

Strong Field Physics

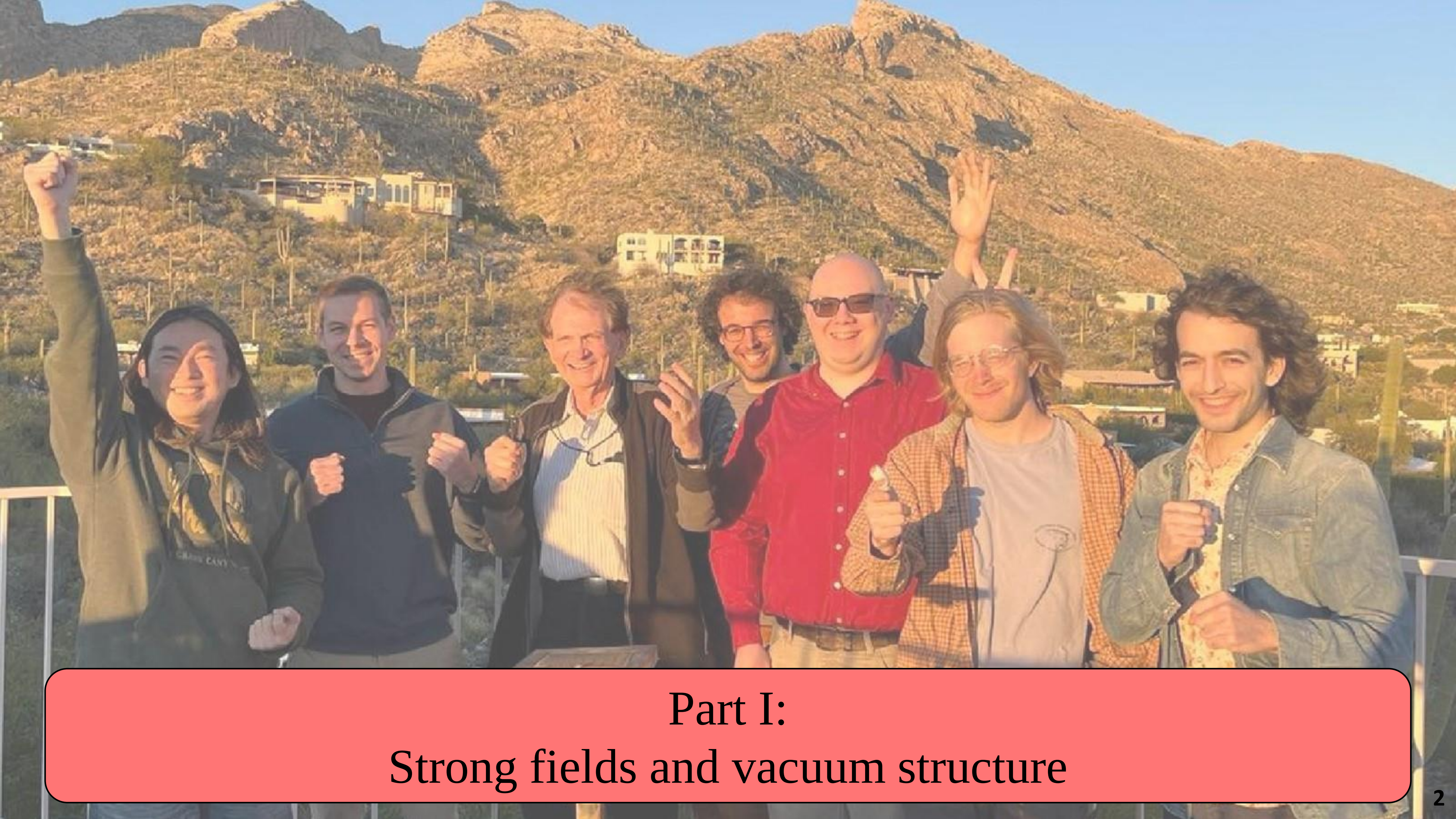
Report on Frankfurt School and current Arizona effort

Prepared by

Johann Rafelski & Andrew Steinmetz

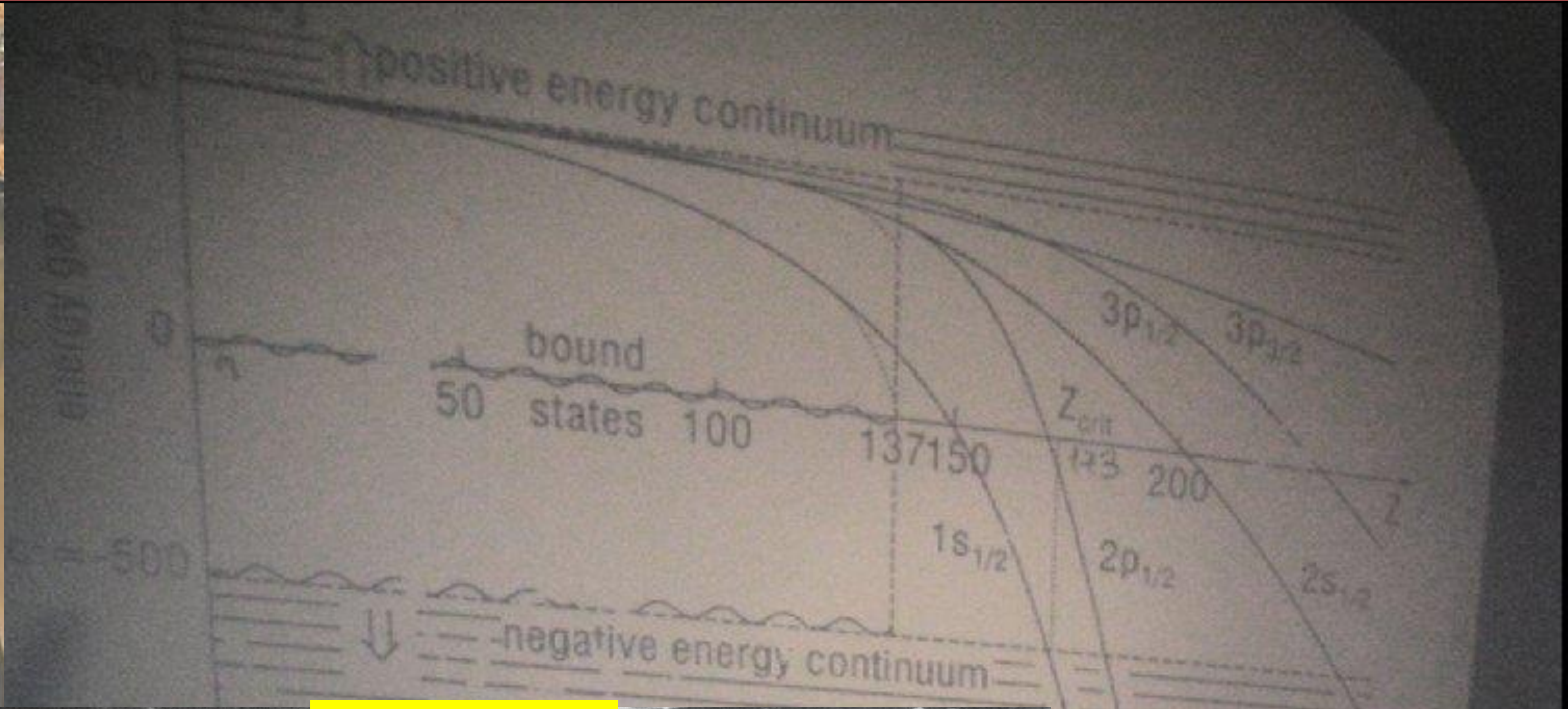
Margaret Island Symposium 2022 on Vacuum Structure, Particles, and Plasmas

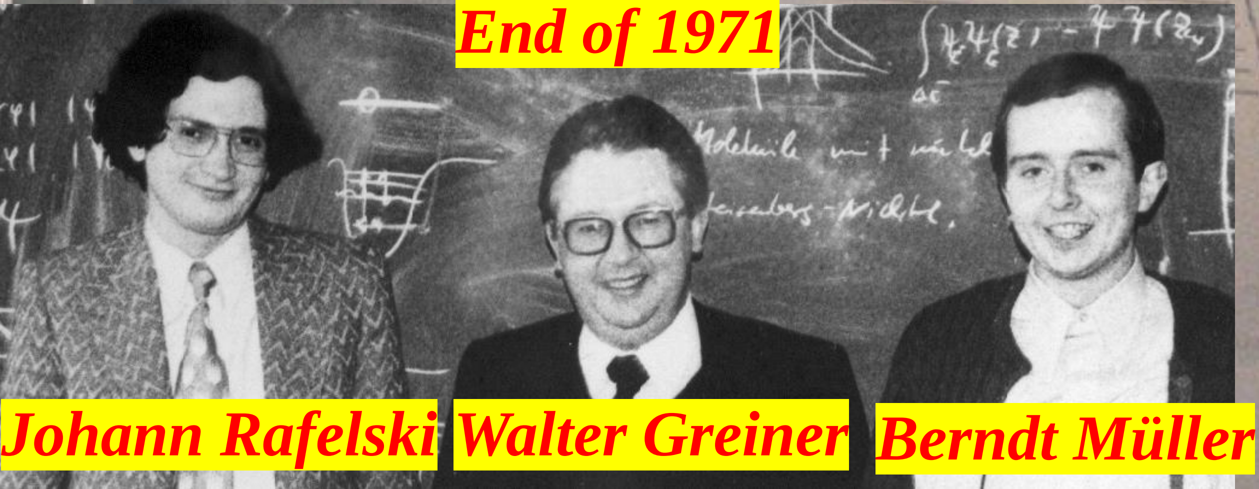
May 17th, 2022



Part I:
Strong fields and vacuum structure

Continuing the Frankfurt School legacy of strong fields physics





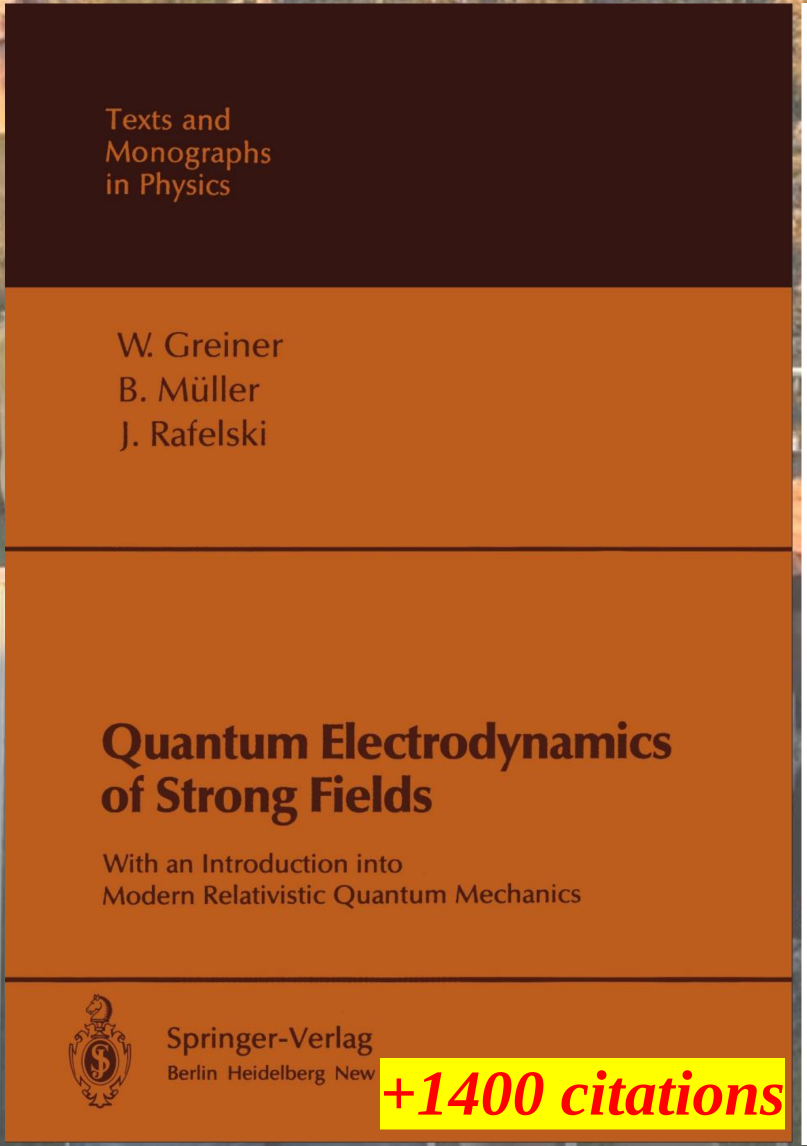
End of 1971

Johann Rafelski ***Walter Greiner*** ***Berndt Müller***

UCLA, 2006

Continuing the Frankfurt School legacy of QED strong field physics

Report of strong fields physics: 1986


Texts and
Monographs
in Physics

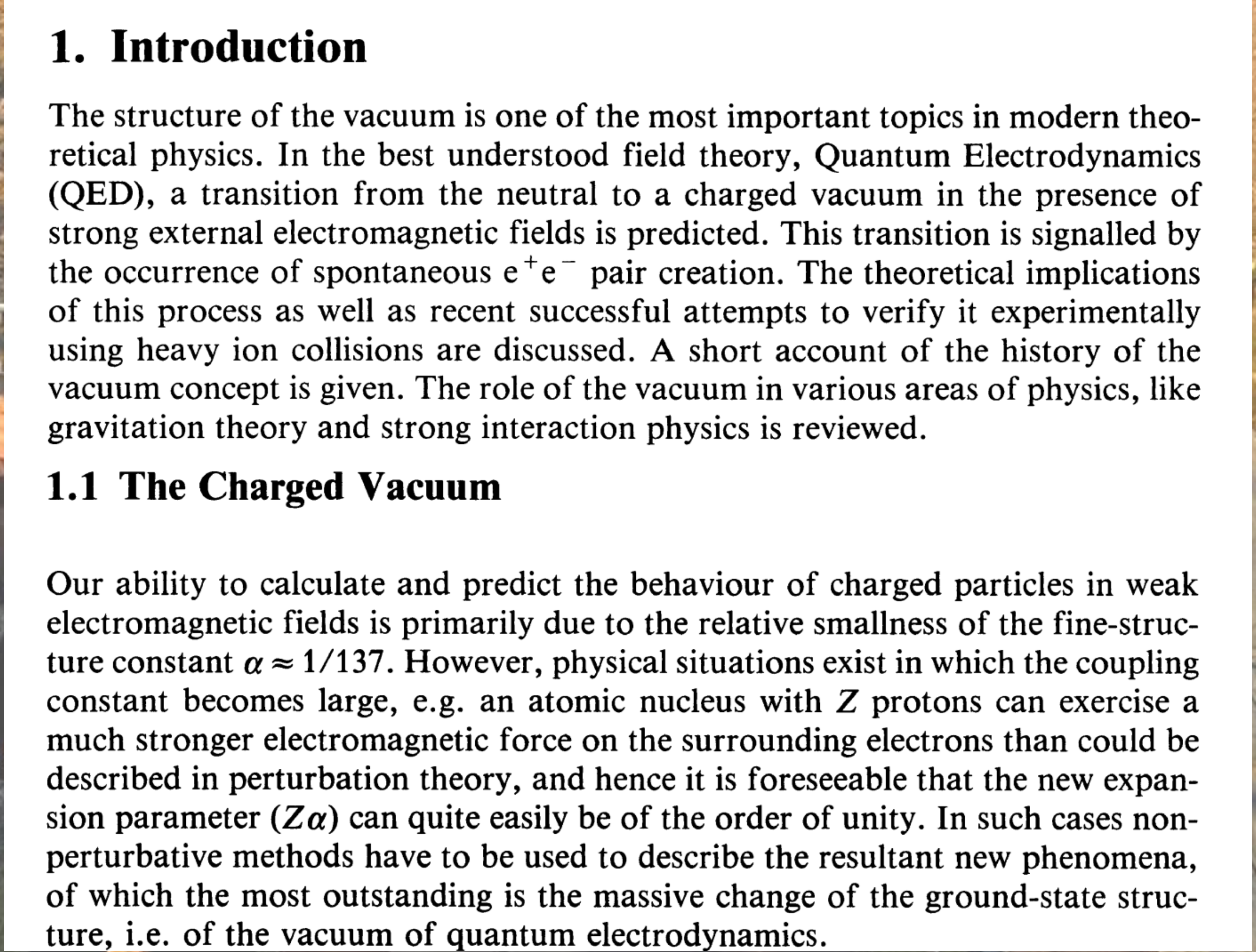
W. Greiner
B. Müller
J. Rafelski

Quantum Electrodynamics
of Strong Fields

With an Introduction into
Modern Relativistic Quantum Mechanics

Springer-Verlag
Berlin Heidelberg New

+1400 citations

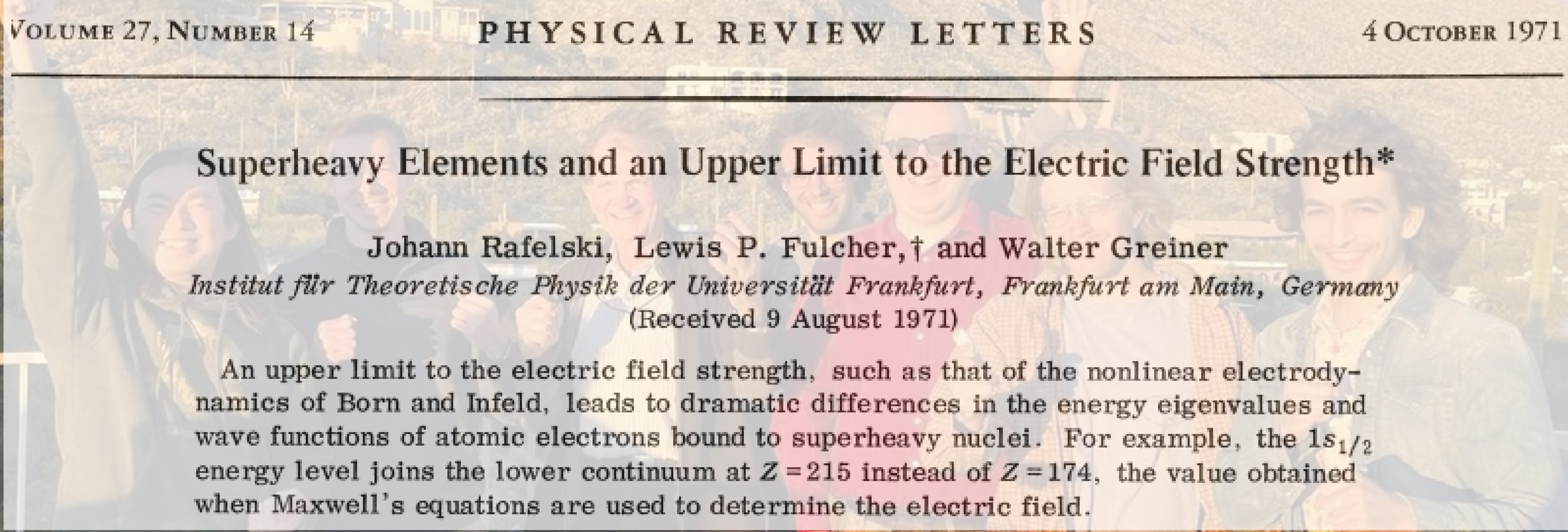

1. Introduction

The structure of the vacuum is one of the most important topics in modern theoretical physics. In the best understood field theory, Quantum Electrodynamics (QED), a transition from the neutral to a charged vacuum in the presence of strong external electromagnetic fields is predicted. This transition is signalled by the occurrence of spontaneous e^+e^- pair creation. The theoretical implications of this process as well as recent successful attempts to verify it experimentally using heavy ion collisions are discussed. A short account of the history of the vacuum concept is given. The role of the vacuum in various areas of physics, like gravitation theory and strong interaction physics is reviewed.

1.1 The Charged Vacuum

Our ability to calculate and predict the behaviour of charged particles in weak electromagnetic fields is primarily due to the relative smallness of the fine-structure constant $\alpha \approx 1/137$. However, physical situations exist in which the coupling constant becomes large, e.g. an atomic nucleus with Z protons can exercise a much stronger electromagnetic force on the surrounding electrons than could be described in perturbation theory, and hence it is foreseeable that the new expansion parameter $(Z\alpha)$ can quite easily be of the order of unity. In such cases non-perturbative methods have to be used to describe the resultant new phenomena, of which the most outstanding is the massive change of the ground-state structure, i.e. of the vacuum of quantum electrodynamics.

My 50^{th}_{10} year anniversary of strong fields physics and limiting acceleration publications!



VOLUME 27, NUMBER 14 PHYSICAL REVIEW LETTERS 4 OCTOBER 1971

Superheavy Elements and an Upper Limit to the Electric Field Strength*

Johann Rafelski, Lewis P. Fulcher,† and Walter Greiner
Institut für Theoretische Physik der Universität Frankfurt, Frankfurt am Main, Germany

(Received 9 August 1971)

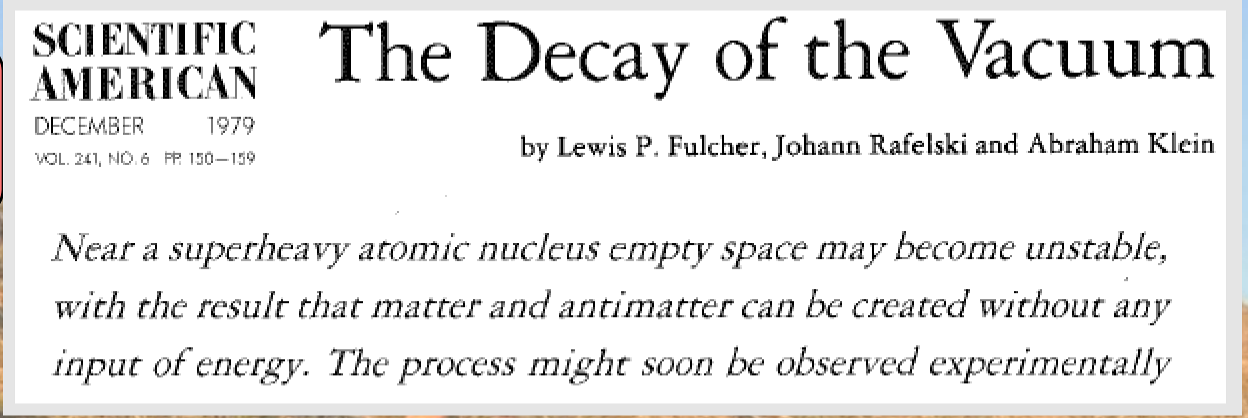
An upper limit to the electric field strength, such as that of the nonlinear electrodynamics of Born and Infeld, leads to dramatic differences in the energy eigenvalues and wave functions of atomic electrons bound to superheavy nuclei. For example, the $1s_{1/2}$ energy level joins the lower continuum at $Z = 215$ instead of $Z = 174$, the value obtained when Maxwell's equations are used to determine the electric field.

One can also celebrate my 60^{th}_{12} birthday in base 12.

We will not celebrate my 200^{th}_{6} year in base 6. (Private communication: Tomás Biró)

J. Rafelski, L. P. Fulcher, and W. Greiner. "Superheavy elements and an upper limit to the electric field strength." Physical Review Letters 27.14 (1971): 958.

First local vacuum structure model: Strong fields and charged vacuum: 1973

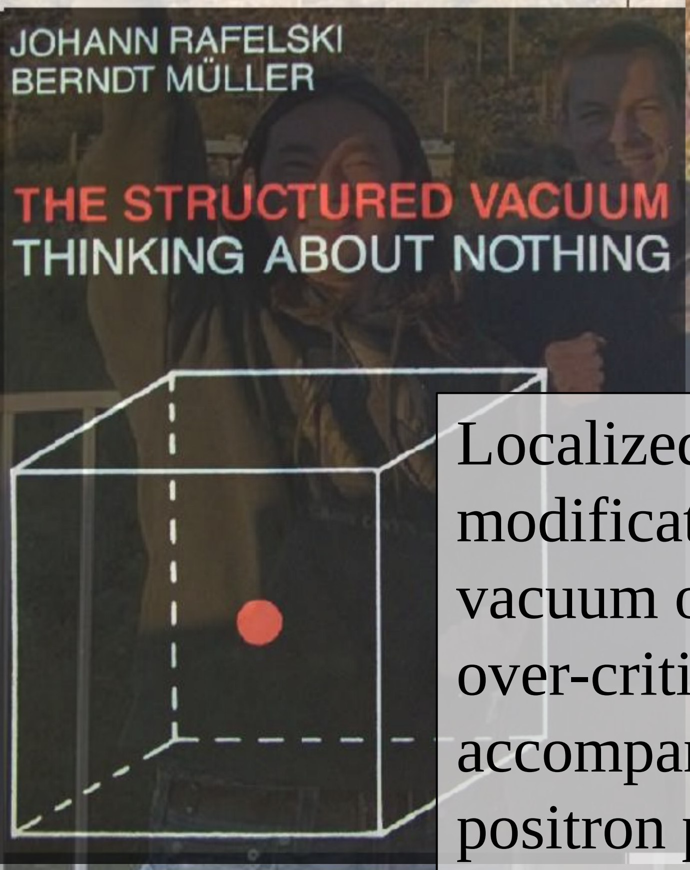


SCIENTIFIC AMERICAN
DECEMBER 1979
VOL. 241, NO. 6 PP. 150—159

The Decay of the Vacuum

by Lewis P. Fulcher, Johann Rafelski and Abraham Klein

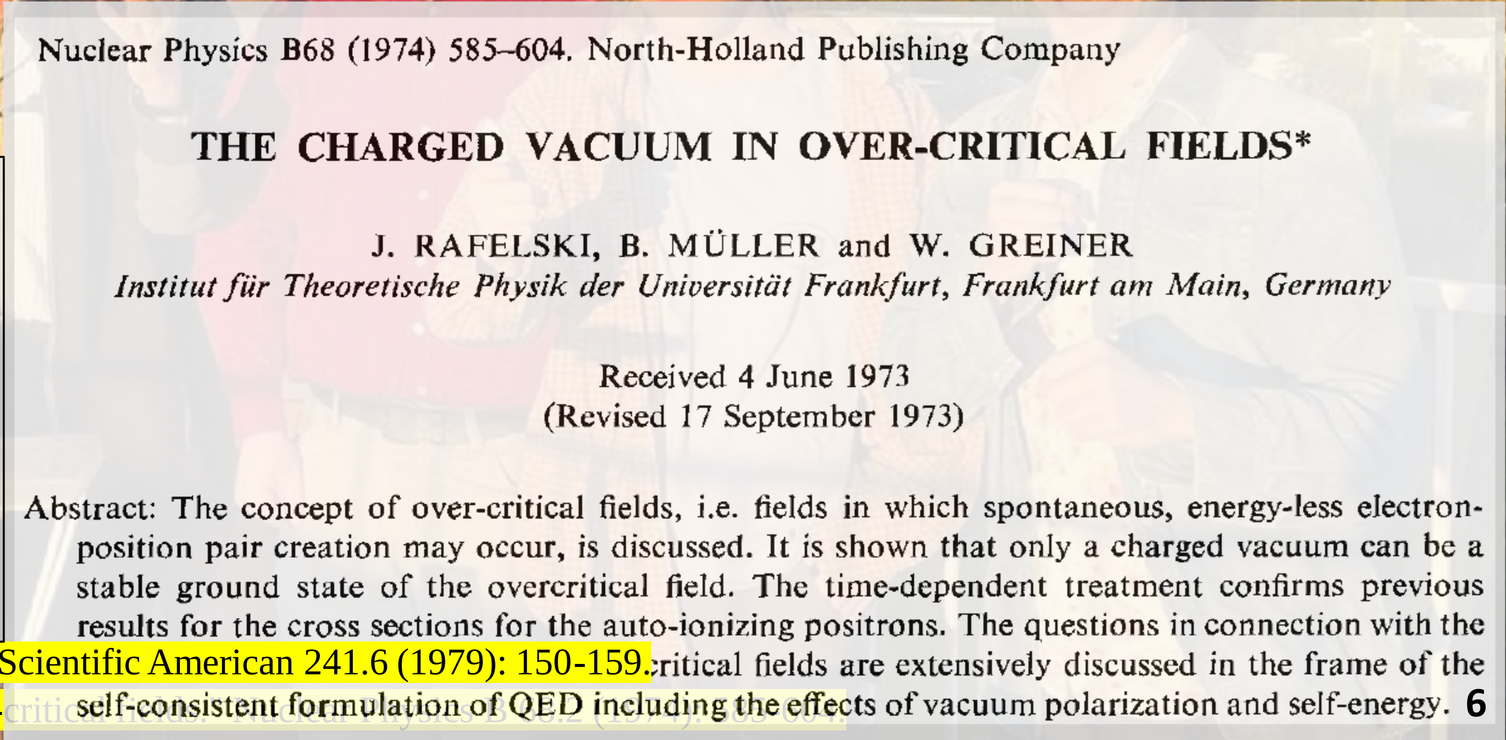
Near a superheavy atomic nucleus empty space may become unstable, with the result that matter and antimatter can be created without any input of energy. The process might soon be observed experimentally



JOHANN RAFELSKI
BERNDT MÜLLER

THE STRUCTURED VACUUM
THINKING ABOUT NOTHING

Localized modification to the vacuum occurs in over-critical fields accompanied by positron production



Nuclear Physics B68 (1974) 585–604. North-Holland Publishing Company

THE CHARGED VACUUM IN OVER-CRITICAL FIELDS*

J. RAFELSKI, B. MÜLLER and W. GREINER
Institut für Theoretische Physik der Universität Frankfurt, Frankfurt am Main, Germany

Received 4 June 1973
(Revised 17 September 1973)

Abstract: The concept of over-critical fields, i.e. fields in which spontaneous, energy-less electron-position pair creation may occur, is discussed. It is shown that only a charged vacuum can be a stable ground state of the overcritical field. The time-dependent treatment confirms previous results for the cross sections for the auto-ionizing positrons. The questions in connection with the critical fields are extensively discussed in the frame of the self-consistent formulation of QED including the effects of vacuum polarization and self-energy.

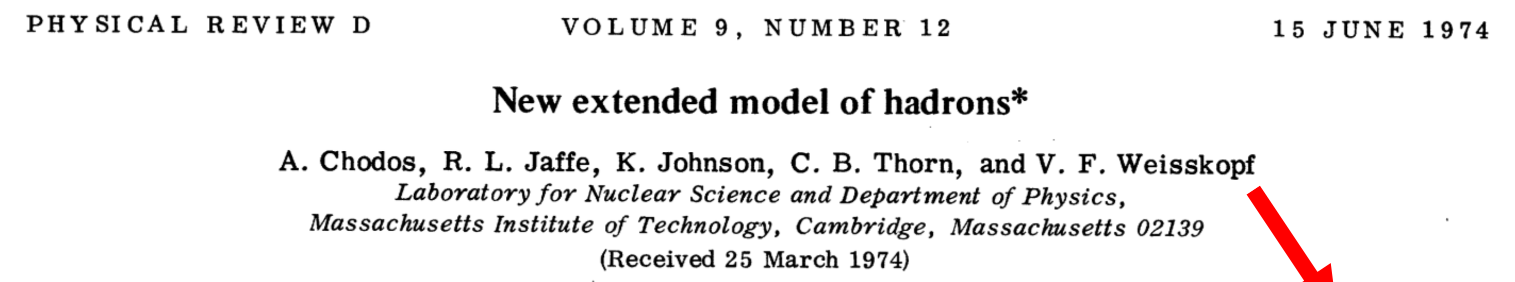
L. P. Fulcher, J. Rafelski, and A. Klein. "The Decay of the Vacuum." Scientific American 241.6 (1979): 150-159.

J. Rafelski, B. Müller, and W. Greiner. "The charged vacuum in over-critical fields." Nuclear Physics B 68.2 (1974): 585-604.

Application of local vacuum structure model to quark confinement inside hadrons: 1974

- Quarks live inside a domain where the (perturbative) vacuum is without gluon fluctuations. This outside structure wants to enter but is kept away by the quarks trying to escape: $P_{quark} = B$.

- The inside of the hadron is an excited state where the energy density is ${}^{E}/_{V} = B$.

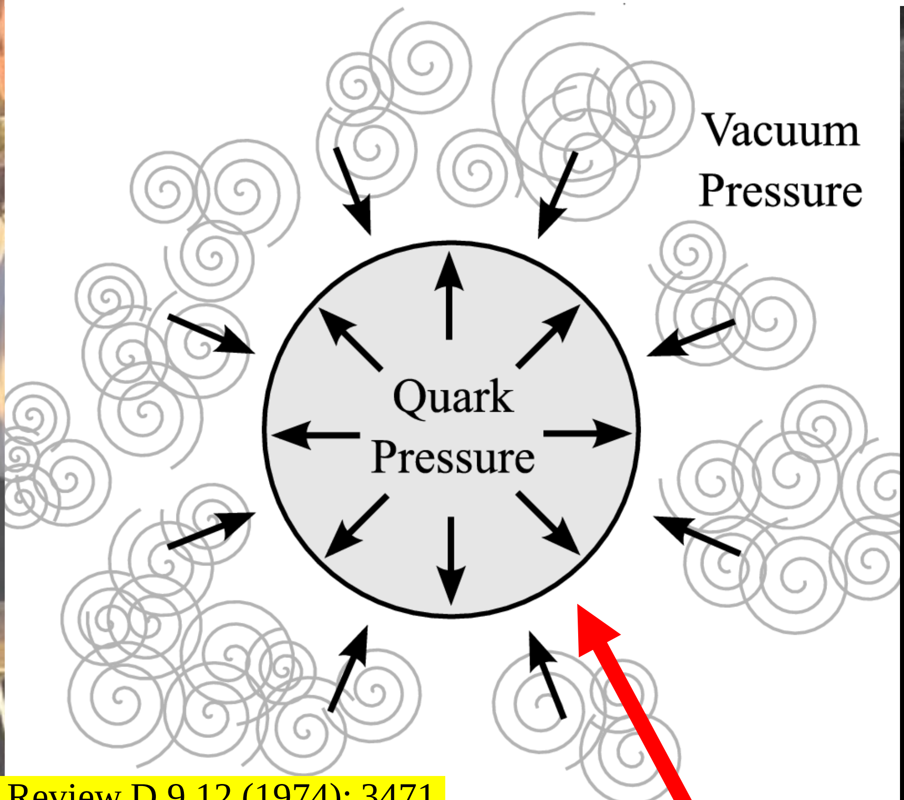


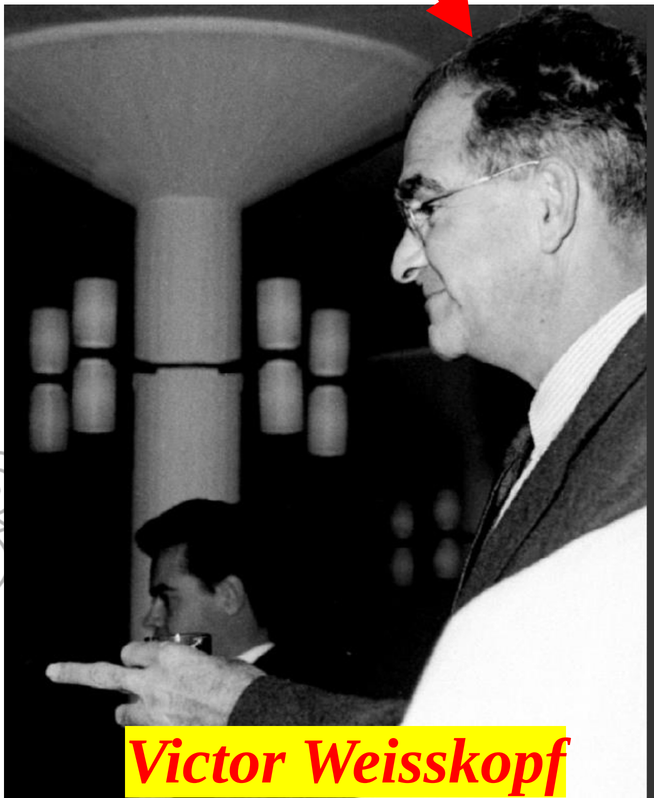
PHYSICAL REVIEW D VOLUME 9, NUMBER 12 15 JUNE 1974

New extended model of hadrons*

A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf
Laboratory for Nuclear Science and Department of Physics,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 25 March 1974)





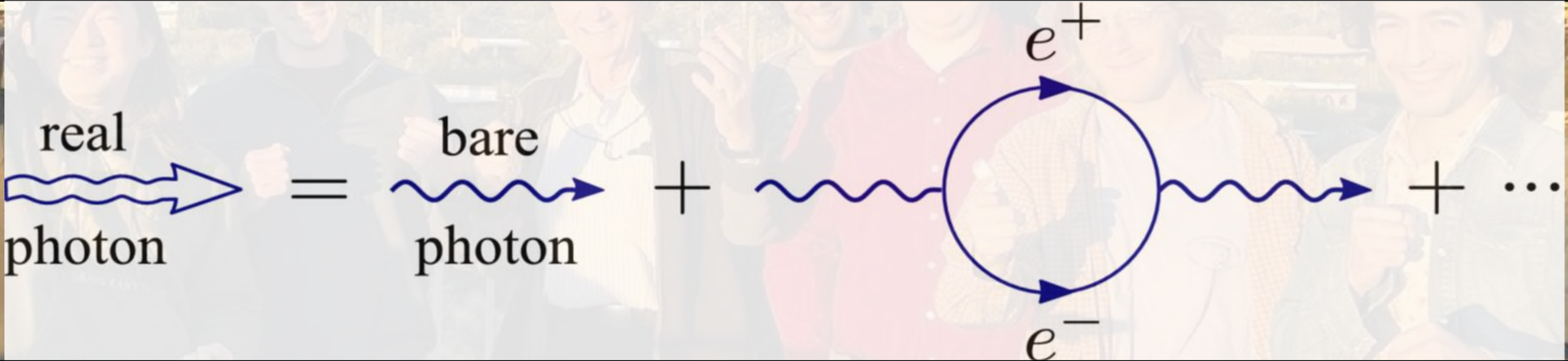
Victor Weisskopf

A. Chodos, et al. "New extended model of hadrons." Physical Review D 9.12 (1974): 3471.

V. Weisskopf, "The electrodynamics of the vacuum based on the quantum theory of the electron," Kong. Dan. Vid. Sel. Mat. Fys. Med. 14, N6, 1 (1936).

Virtual pairs: The vacuum is a dielectric

The vacuum is a dielectric medium as charges are screened by particle-hole (pair) excitations. In Feynman’s language the real photon is decomposed into a bare photon and a photon turning into a “virtual” pair. The result: renormalized electron charge smaller than bare. The observable Coulomb interaction stronger (0.4%) at distance $\hbar c/m_e$

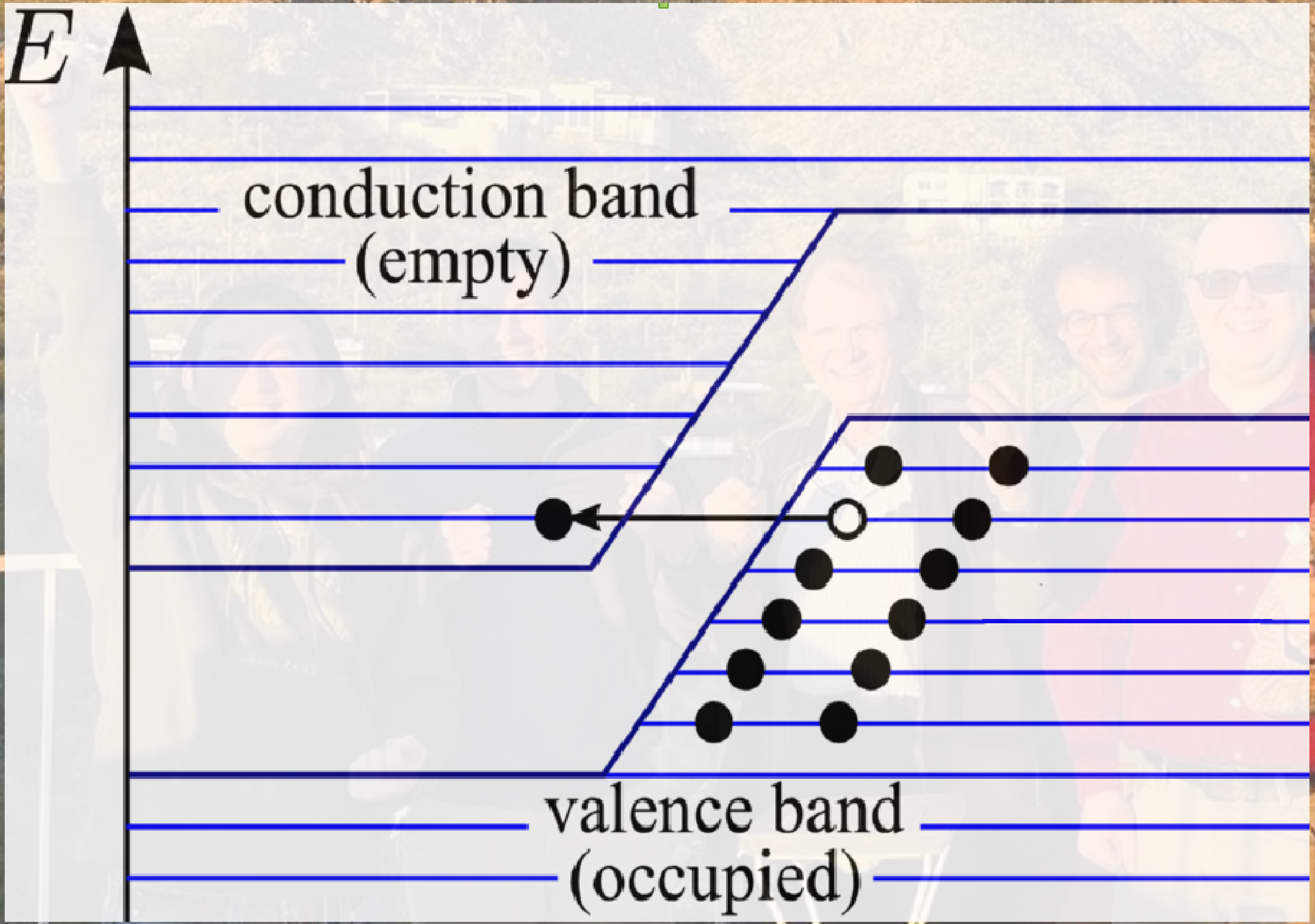


This effect has been studied in depth in atomic physics and is of particular relevance for exotic atoms where a heavy (muon) charged particle replaces an electron.

See next presentation by Martin Formanek for polarization phenomena.

E. A. Uehling, “Polarization effects in the positron theory.” Physical Review 48.1 (1935): 55.

Vacuum decay: Pair production instability



- Relativistic Dirac quantum physics **predicts antimatter** and allows for the formation of pairs of particles and antiparticles.
- The relativistic gap in energy is reminiscent of insulators where the conductive band is above the valance (occupied) electron band.

Sauter step model will be discussed in Stefan Evan's talk.

F. Sauter, "Zum 'Kleinschen Paradoxon'," Z. Phys. 73 (1932), 547-552 doi:10.1007/BF01349862
S. Evans, and J. Rafelski. "Particle production at a finite potential step: transition from Euler–Heisenberg to Klein paradox." The European Physical Journal A 57.12 (2021): 1-10.

The sparking of the QED vacuum in quasi-constant fields

All E-fields are unstable and can decay into particles if energy is available and rate is large enough.

– Explained by Heisenberg in 1935 and by Schwinger's article in 1950 appearing almost an after thought. (*my idea how this happened: invited by referee=Heisenberg*?)

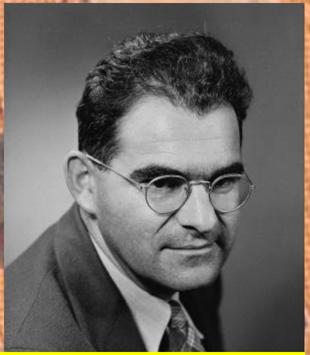
Effect large for E-field:

$$E_{cr} = \frac{\left(m_e c^2\right)^2}{e\hbar c} = 1.323 \times 10^{18}\,\frac{\mathrm{V}}{\mathrm{m}}$$

Persistence probability of the empty vacuum:

$$P \sim \exp\left(-\pi \frac{E_{cr}}{|\boldsymbol{E}|}\right)$$

W. Heisenberg

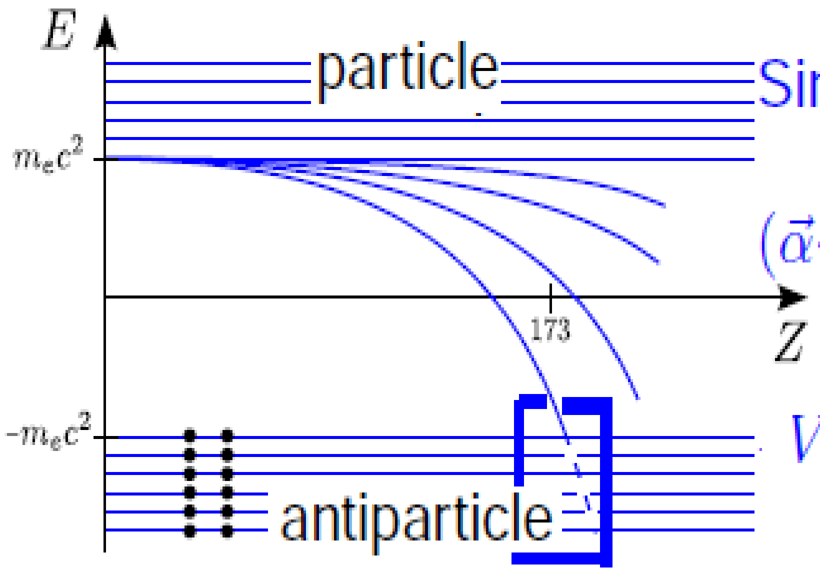


V. Weisskopf

J. Schwinger

See Stefan Evans's presentation for in-depth look at surface pair production.

Vacuum instability in highly charged atomic systems:
What is this (mostly) about?



Single Particle Dirac Equation

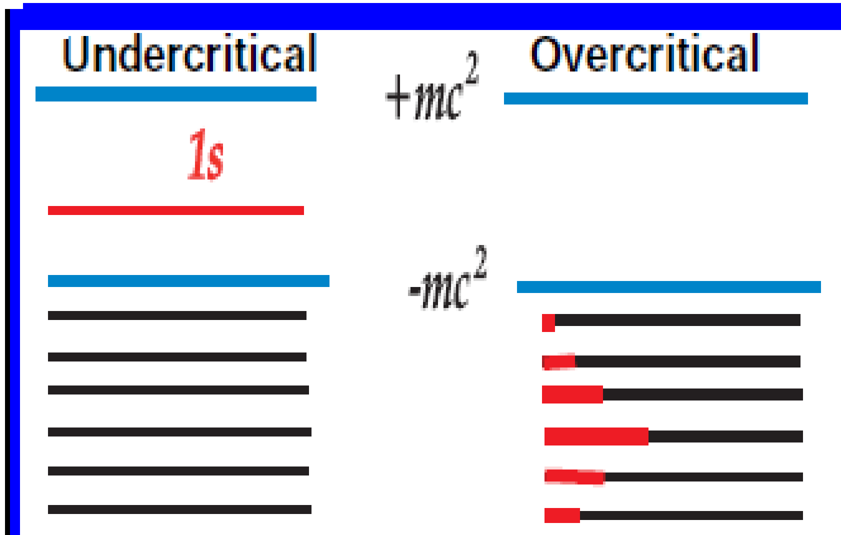
$$(\vec{\alpha}\cdot i\vec{\nabla}+\beta m+V(r))\Psi_n(\vec{r}) = E_n\Psi_n(\vec{r})$$

$$V(r) = \begin{cases} -\dfrac{Z\alpha}{r} & r > R_N \\ -\dfrac{3}{2}\dfrac{Z\alpha}{R_N} + \dfrac{r^2}{2}\dfrac{Z\alpha}{R_N^3} & r < R_N \end{cases}$$

Supercritical fields

The bound states drawn from one continuum move as function of Z across into the other continuum. Mix-up of particle/antiparticle states

Reference: W. Greiner, B. Müller and JR ISBN 3-540-13404-2.
"Quantum Electrodynamics of Strong Fields,"
(Springer Texts and Monographs in Physics, 1985),



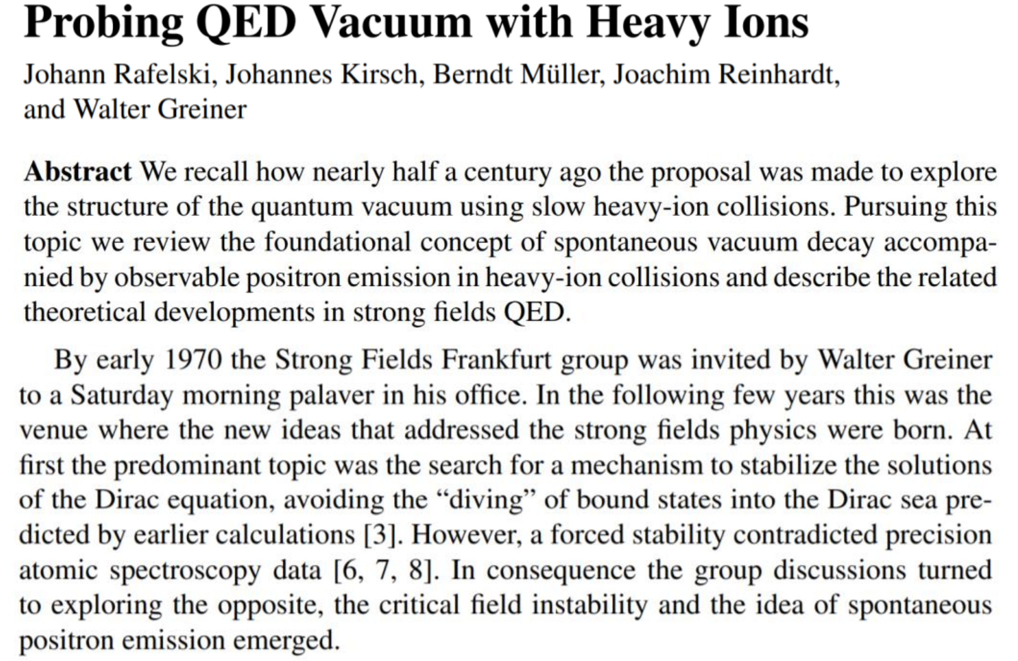
If diving state 'empty' vacuum decays

$|Q=0\rangle \rightarrow |Q=e\rangle + e^+$ by positron

state occupied by an electron, 'smooth

transition of charge distribution

Vacuum instability and strong fields in heavy ion collisions
Multitude of possible dynamical processes

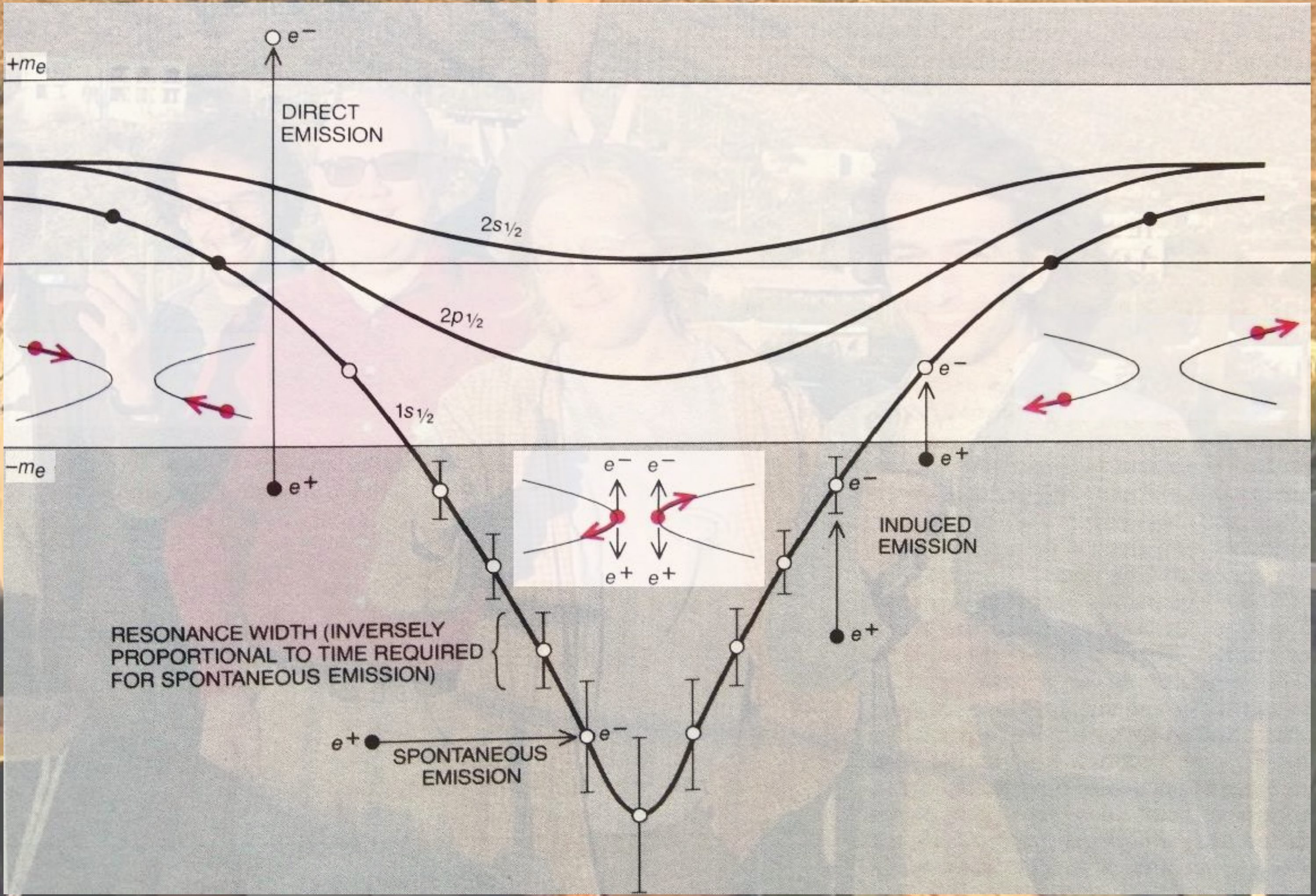


Probing QED Vacuum with Heavy Ions

Johann Rafelski, Johannes Kirsch, Berndt Müller, Joachim Reinhardt,
and Walter Greiner

Abstract We recall how nearly half a century ago the proposal was made to explore the structure of the quantum vacuum using slow heavy-ion collisions. Pursuing this topic we review the foundational concept of spontaneous vacuum decay accompanied by observable positron emission in heavy-ion collisions and describe the related theoretical developments in strong fields QED.

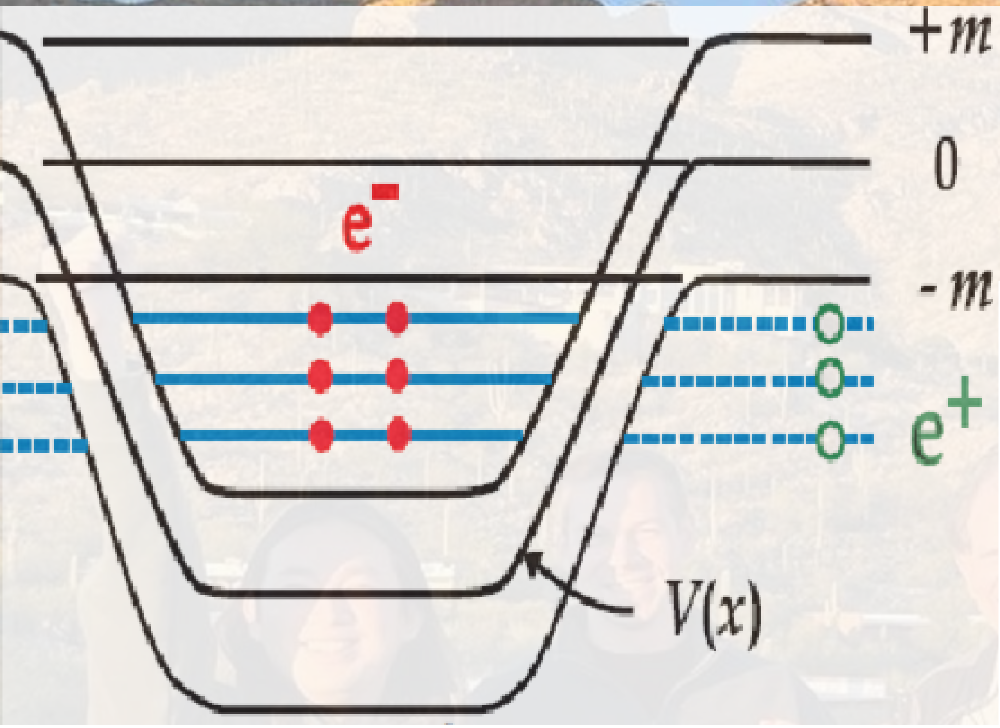
By early 1970 the Strong Fields Frankfurt group was invited by Walter Greiner to a Saturday morning palaver in his office. In the following few years this was the venue where the new ideas that addressed the strong fields physics were born. At first the predominant topic was the search for a mechanism to stabilize the solutions of the Dirac equation, avoiding the "diving" of bound states into the Dirac sea predicted by earlier calculations [3]. However, a forced stability contradicted precision atomic spectroscopy data [6, 7, 8]. In consequence the group discussions turned to exploring the opposite, the critical field instability and the idea of spontaneous positron emission emerged.

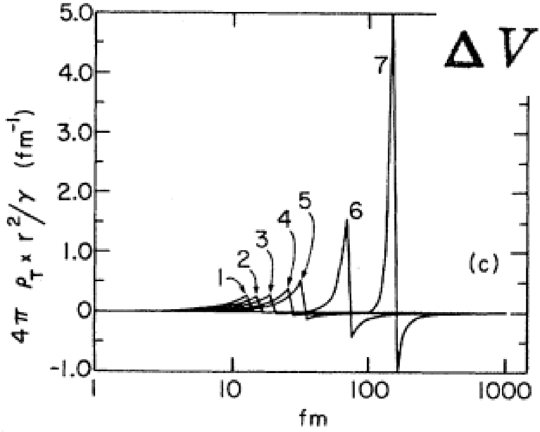


J. Rafelski, et al. "Probing QED vacuum with heavy ions." New horizons in fundamental physics. Springer, Cham, 2017. 211-251.

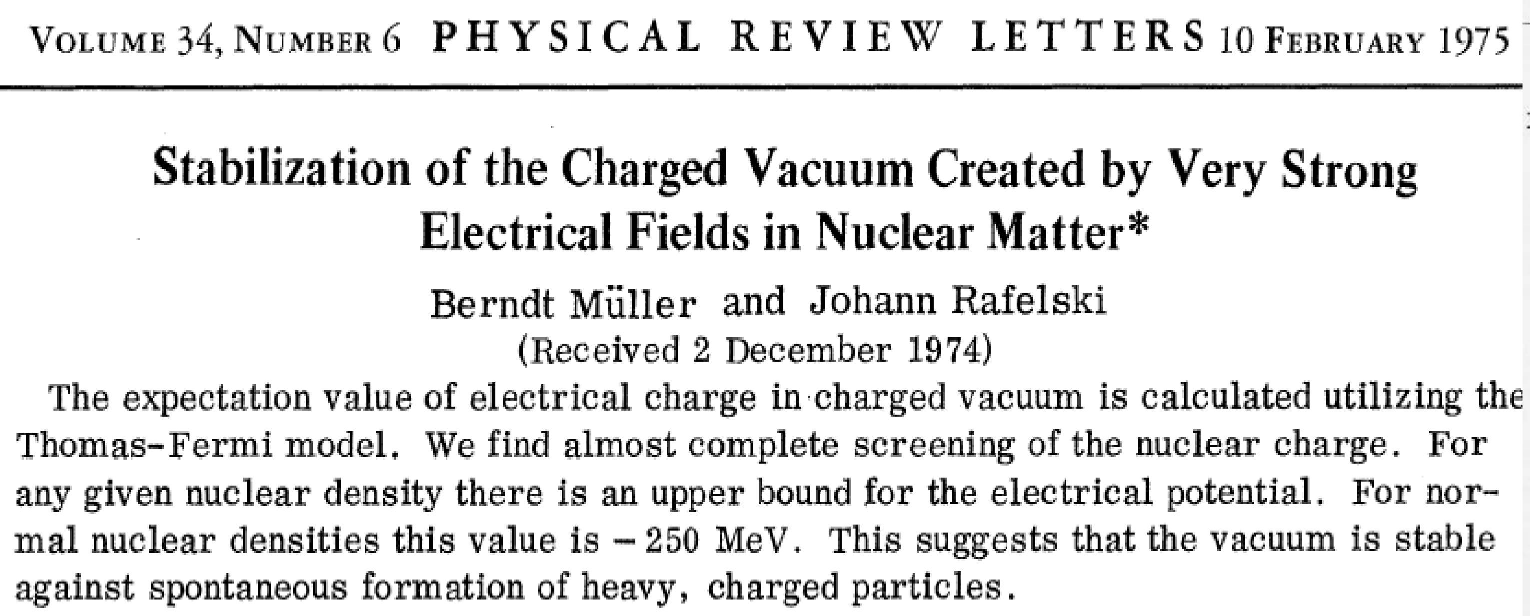
Stabilization of the local vacuum state

Back reaction of accumulated vacuum charge can be accounted for self-consistently.





$$\Delta V(r) = e\rho_N(r) - (e^2/3\pi^2)(2mV + V^2)^{3/2}$$

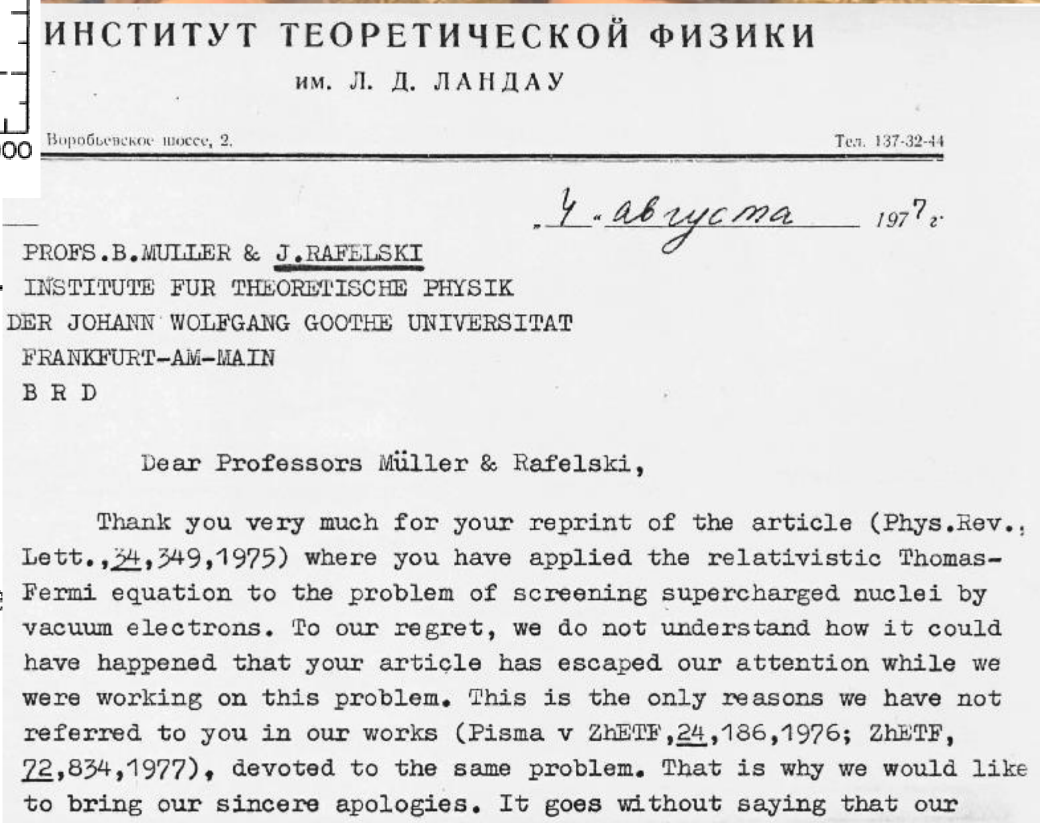

VOLUME 34, NUMBER 6 PHYSICAL REVIEW LETTERS 10 FEBRUARY 1975

Stabilization of the Charged Vacuum Created by Very Strong Electrical Fields in Nuclear Matter*

Berndt Müller and Johann Rafelski

(Received 2 December 1974)

The expectation value of electrical charge in charged vacuum is calculated utilizing the Thomas-Fermi model. We find almost complete screening of the nuclear charge. For any given nuclear density there is an upper bound for the electrical potential. For normal nuclear densities this value is −250 MeV. This suggests that the vacuum is stable against spontaneous formation of heavy, charged particles.


ИНСТИТУТ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ
им. Л. Д. ЛАНДАУ
Воробьевское шоссе, 2. Тел. 137-32-44

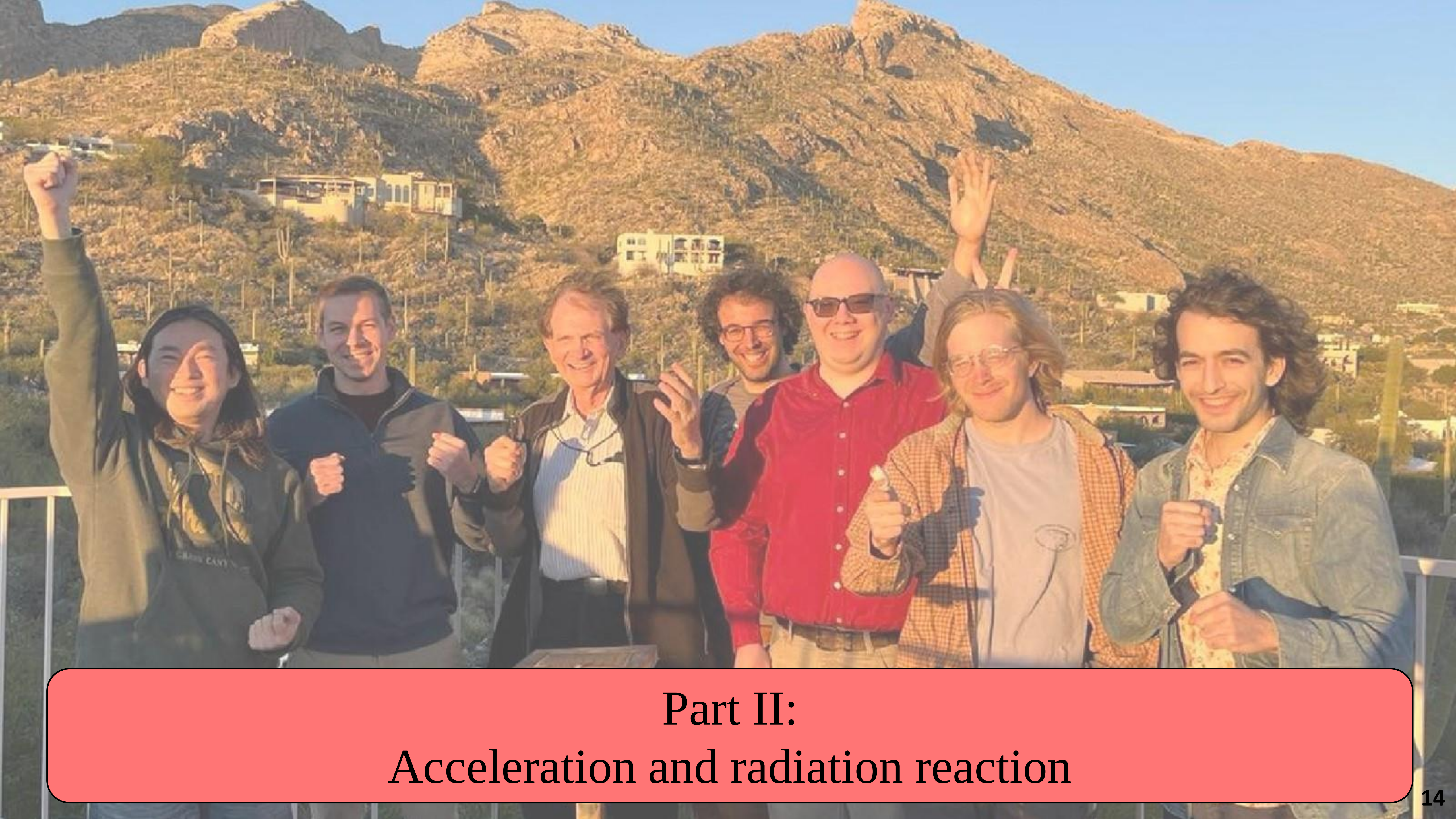
„4" августа 1977 г.

PROFS.B.MULLER & J.RAFELSKI
INSTITUTE FUR THEORETISCHE PHYSIK
DER JOHANN WOLFGANG GOOTHE UNIVERSITAT
FRANKFURT-AM-MAIN
B R D

Dear Professors Müller & Rafelski,

Thank you very much for your reprint of the article (Phys.Rev., Lett.,34,349,1975) where you have applied the relativistic Thomas-Fermi equation to the problem of screening supercharged nuclei by vacuum electrons. To our regret, we do not understand how it could have happened that your article has escaped our attention while we were working on this problem. This is the only reasons we have not referred to you in our works (Pisma v ZhETF,24,186,1976; ZhETF, 72,834,1977), devoted to the same problem. That is why we would like to bring our sincere apologies. It goes without saying that our

B. Müller, and J. Rafelski. "Stabilization of the charged vacuum created by very strong electrical fields in nuclear matter." Physical Review Letters 34.6 (1975): 349.



Part II:
Acceleration and radiation reaction

A few remarks about Planck acceleration

As per Wikipedia, the definition of "true" Planckian acceleration is

$$a_{Pl} = \frac{c^2}{\ell_{Pl}} = 5.55 \times 10^{51}\,\frac{\text{m}}{\text{s}^2} \qquad \ell_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \qquad M_{Pl} = \sqrt{\frac{G\hbar}{c}}$$

However, we define critical acceleration via the Compton wavelength λ_C as

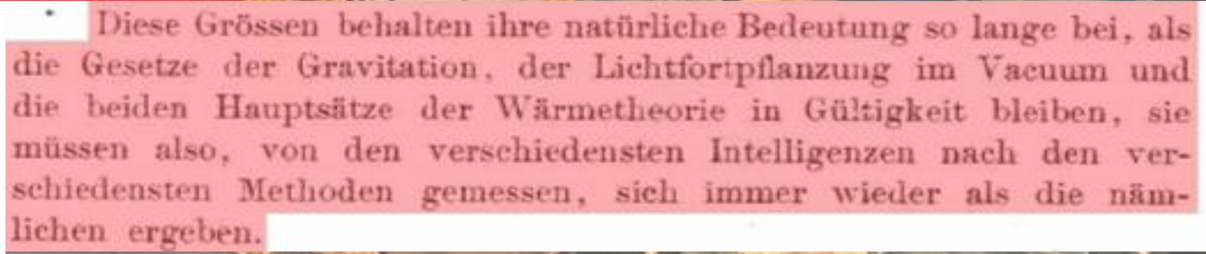
$$a_{cr} = \frac{c^2}{\lambda_C} = m_e c^2 \frac{c}{\hbar}$$

The appearance of mass clarifies that the critical acceleration is related to particle mass. Replacing m_e by the Planck mass M_{Pl} reproduces a_{Pl}. Critical "Planck" acceleration is the same acceleration felt by two particles due to Newtonian gravity at a distance of ℓ_{Pl}.

$$ma = \frac{Gm^2}{\ell_{Pl}^2} \rightarrow a_{cr} = \frac{Gm_e}{\ell_{Pl}^2} = \frac{c^2}{\lambda_C} = 2.33 \times 10^{29}\,\frac{\text{m}}{\text{s}^2}$$

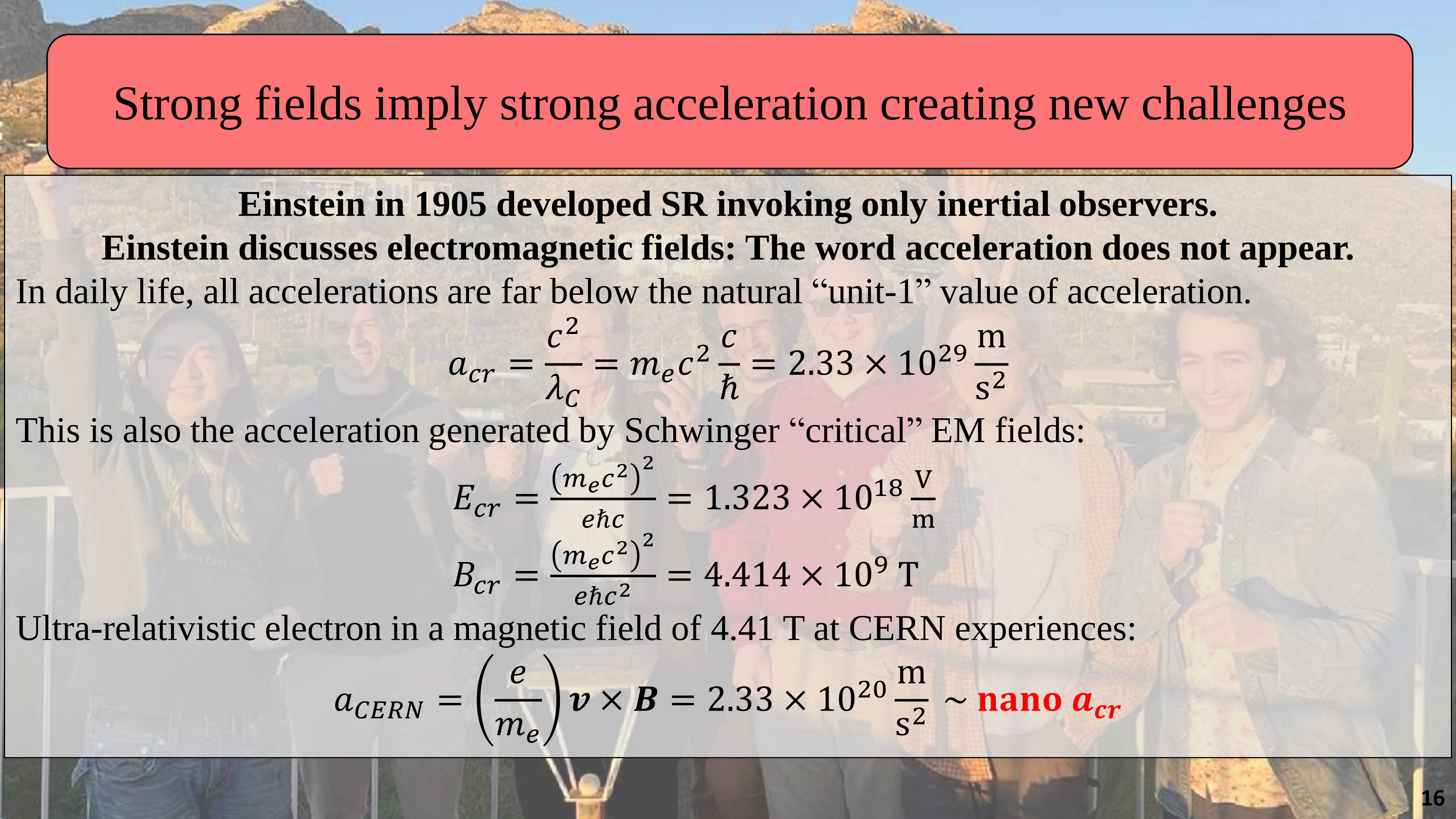
We call study of the critical acceleration a_{cr} domain the **Acceleration Frontier** of which the lowest accessible case is that of the electron.

"These scales retain their natural meaning as long as the law of gravitation, the velocity of light in vacuum and the central equations of thermodynamics remain valid, and therefore they must always arise, among different intelligences employing different means of measuring." *M. Planck, "Über irreversible Strahlungsvorgänge." Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* **5**, *440-480 (1899), (last page)*



Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Strong fields imply strong acceleration creating new challenges



Einstein in 1905 developed SR invoking only inertial observers.

Einstein discusses electromagnetic fields: The word acceleration does not appear.

In daily life, all accelerations are far below the natural “unit-1” value of acceleration.

$$a_{cr} = \frac{c^2}{\lambda_C} = m_e c^2 \frac{c}{\hbar} = 2.33 \times 10^{29} \, \frac{\mathrm{m}}{\mathrm{s}^2}$$

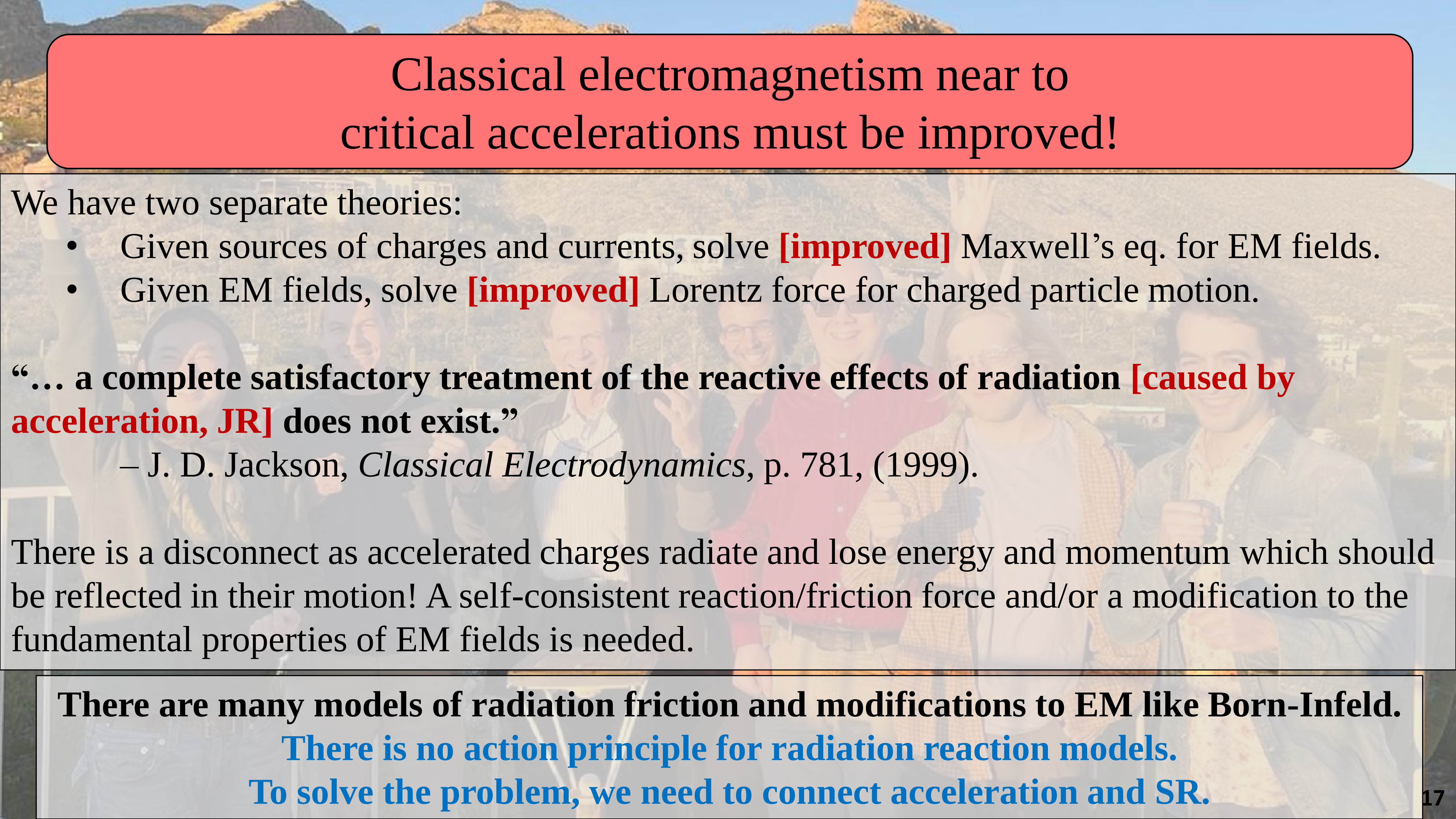
This is also the acceleration generated by Schwinger “critical” EM fields:

$$E_{cr} = \frac{(m_e c^2)^2}{e\hbar c} = 1.323 \times 10^{18} \, \frac{\mathrm{V}}{\mathrm{m}}$$

$$B_{cr} = \frac{(m_e c^2)^2}{e\hbar c^2} = 4.414 \times 10^{9} \ \mathrm{T}$$

Ultra-relativistic electron in a magnetic field of 4.41 T at CERN experiences:

$$a_{CERN} = \left(\frac{e}{m_e}\right) \boldsymbol{v} \times \boldsymbol{B} = 2.33 \times 10^{20} \, \frac{\mathrm{m}}{\mathrm{s}^2} \sim \textbf{nano } \boldsymbol{a_{cr}}$$



Classical electromagnetism near to critical accelerations must be improved!

We have two separate theories:

- Given sources of charges and currents, solve **[improved]** Maxwell’s eq. for EM fields.
- Given EM fields, solve **[improved]** Lorentz force for charged particle motion.

“… a complete satisfactory treatment of the reactive effects of radiation [caused by acceleration, JR] does not exist.”

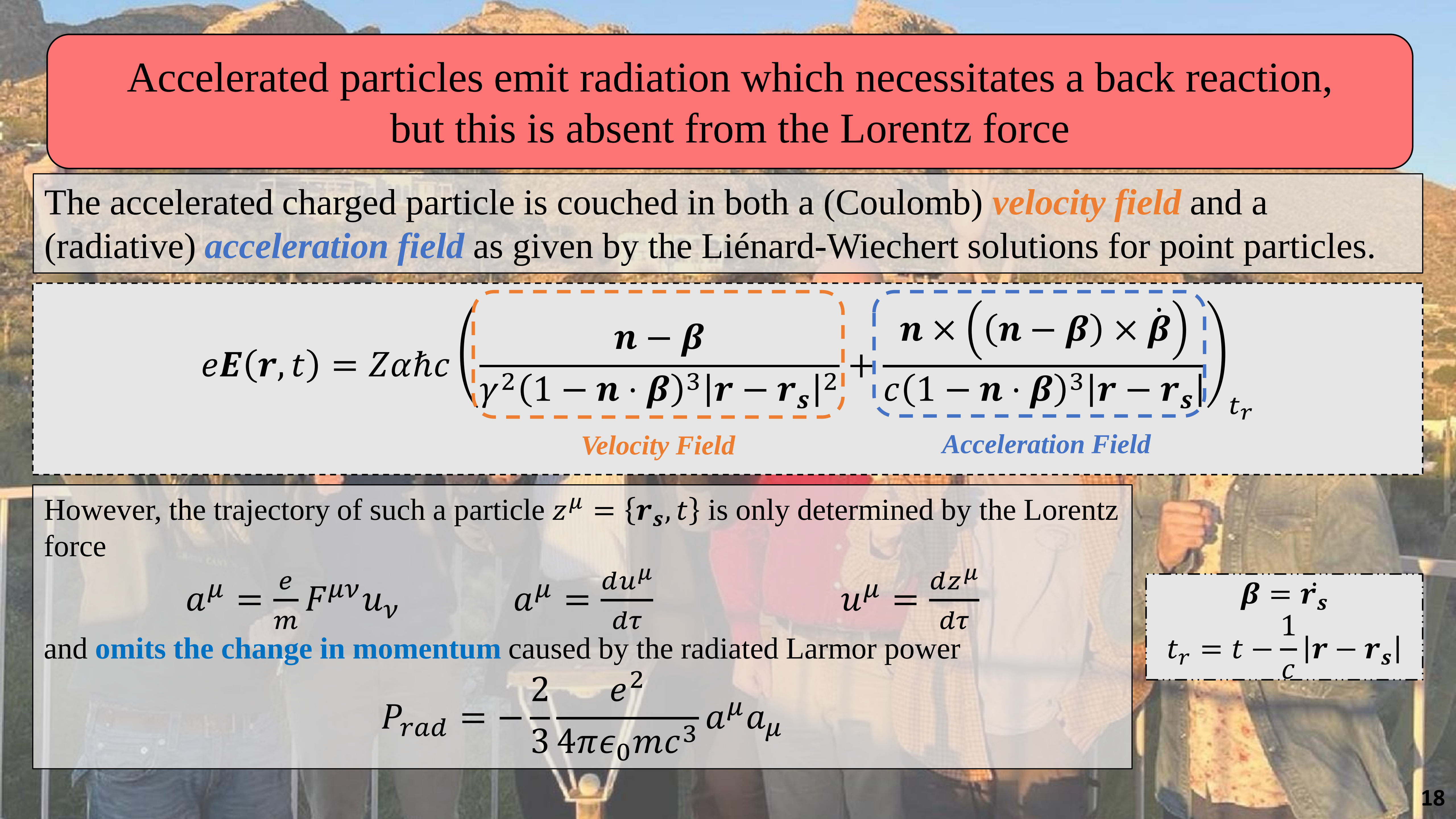
– J. D. Jackson, *Classical Electrodynamics*, p. 781, (1999).

There is a disconnect as accelerated charges radiate and lose energy and momentum which should be reflected in their motion! A self-consistent reaction/friction force and/or a modification to the fundamental properties of EM fields is needed.

There are many models of radiation friction and modifications to EM like Born-Infeld.

There is no action principle for radiation reaction models.

To solve the problem, we need to connect acceleration and SR.



Accelerated particles emit radiation which necessitates a back reaction, but this is absent from the Lorentz force

The accelerated charged particle is couched in both a (Coulomb) ***velocity field*** and a (radiative) ***acceleration field*** as given by the Liénard-Wiechert solutions for point particles.

$$e\boldsymbol{E}(\boldsymbol{r},t) = Z\alpha\hbar c\left(\underbrace{\frac{\boldsymbol{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3|\boldsymbol{r}-\boldsymbol{r_s}|^2}}_{\textit{Velocity Field}} + \underbrace{\frac{\boldsymbol{n}\times\left((\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right)}{c(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3|\boldsymbol{r}-\boldsymbol{r_s}|}}_{\textit{Acceleration Field}}\right)_{t_r}$$

However, the trajectory of such a particle $z^\mu = \{\boldsymbol{r_s}, t\}$ is only determined by the Lorentz force

$$a^\mu = \frac{e}{m}F^{\mu\nu}u_\nu \qquad a^\mu = \frac{du^\mu}{d\tau} \qquad u^\mu = \frac{dz^\mu}{d\tau}$$

and **omits the change in momentum** caused by the radiated Larmor power

$$P_{rad} = -\frac{2}{3}\frac{e^2}{4\pi\epsilon_0 mc^3}a^\mu a_\mu$$

$$\boldsymbol{\beta} = \dot{\boldsymbol{r}}_{\boldsymbol{s}}$$

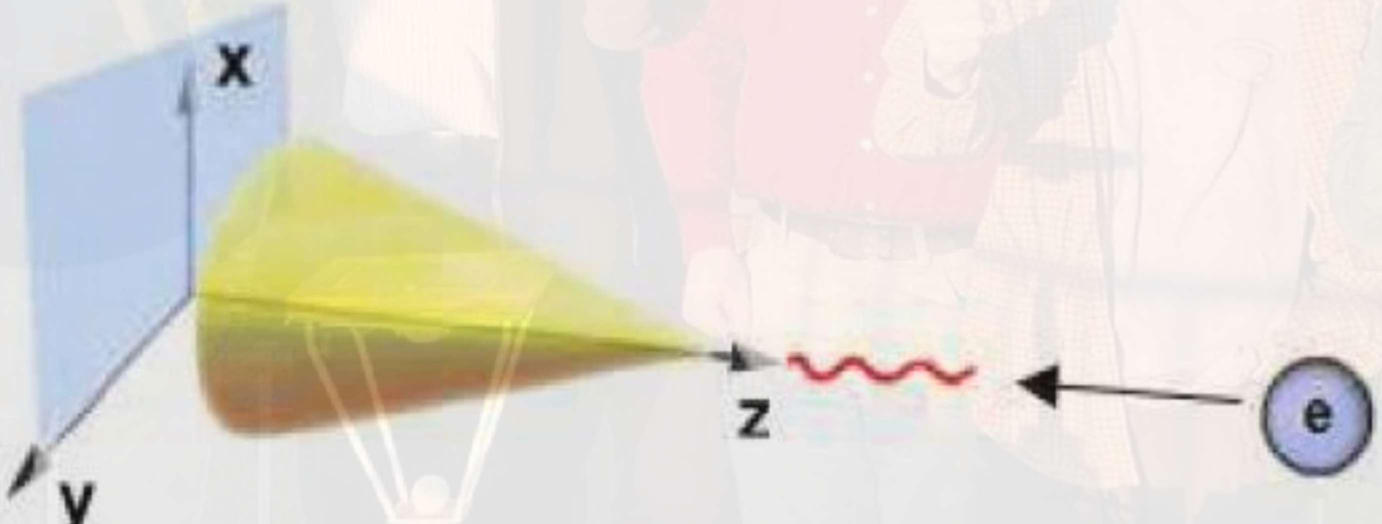
$$t_r = t - \frac{1}{c}|\boldsymbol{r}-\boldsymbol{r_s}|$$

Probing super-critical (Planck) acceleration

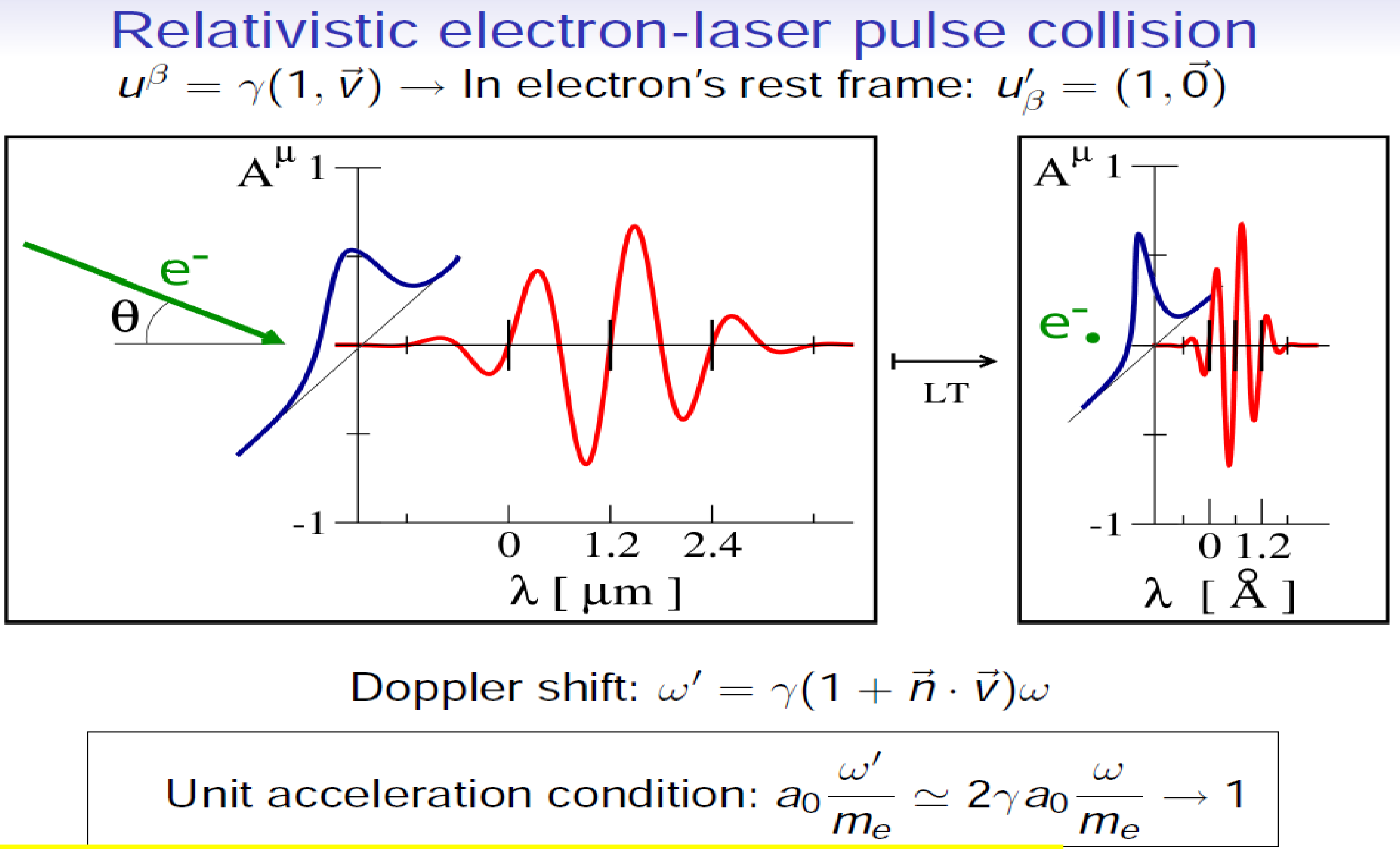
$$a_c = 1(\rightarrow m_e c^3/\hbar = 2.331 \times 10^{29}\mathrm{m/s^2})$$

Plan A: Directly laser accelerate electrons from rest, requires Schwinger scale field and may not be realizable – backreaction and far beyond today's laser pulse intensity technology.

Plan B: Ultra-relativistic <u>Lorentz-boost</u>: we collide counter-propagating electron and laser pulse.



Lorentz transformation (LT) of a light pulse



Relativistic electron-laser pulse collision

$u^\beta = \gamma(1,\vec{v}) \longrightarrow$ In electron's rest frame: $u'_\beta = (1,\vec{0})$

Doppler shift: $\omega' = \gamma(1+\vec{n}\cdot\vec{v})\omega$

Unit acceleration condition: $a_0 \frac{\omega'}{m_e} \simeq 2\gamma a_0 \frac{\omega}{m_e} \longrightarrow 1$

Y. Hadad, et al. "Effects of radiation reaction in relativistic laser acceleration." Physical Review D 82.9 (2010): 096012.

Maybe acceleration is not what we think: Connecting temperature and acceleration
Is there an acceleration?

Strong Fields → ***Temperature*** ← ***Acceleration***

Interpretation of external fields as temperature

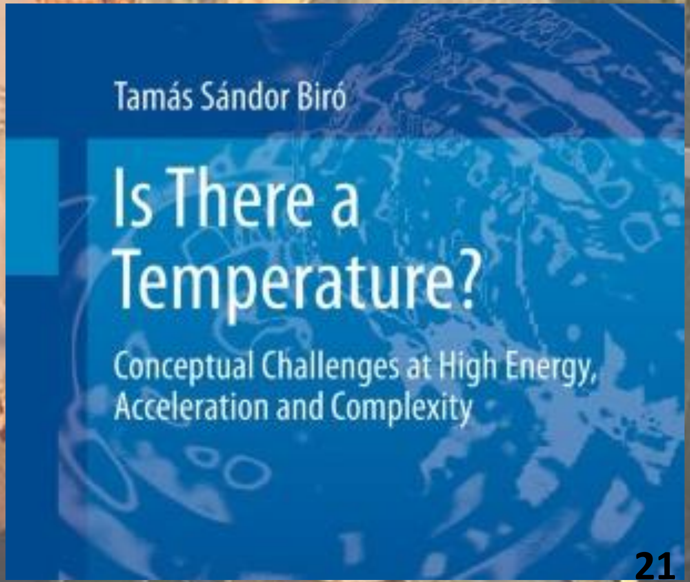
Temperature representation of Euler-Heisenberg action in electric-dominated fields.

Gravity Swing, Taipei 101, (2012)

W. H. Unruh

Notes on black-hole evaporation

Thermal background (Unruh temperature) experienced by an observer undergoing constant acceleration in a field-free vacuum.



Tamás Sándor Biró

Is There a Temperature?

Conceptual Challenges at High Energy, Acceleration and Complexity

B. Müller, W. Greiner, and J. Rafelski. "Interpretation of external fields as temperature.' Physics Letters A 63.3 (1977)

W. G. Unruh, "Notes on black-hole evaporation." Physical Review D 14.4 (1976)

L. Labun and J. Rafelski, "Acceleration and vacuum temperature." Phys. Rev. D 86, 041701(R) (2012)

Part III:
A short guide to current work as presented at this conference

See Martin Formanek's lecture:

Relativistic linear response to EM fields

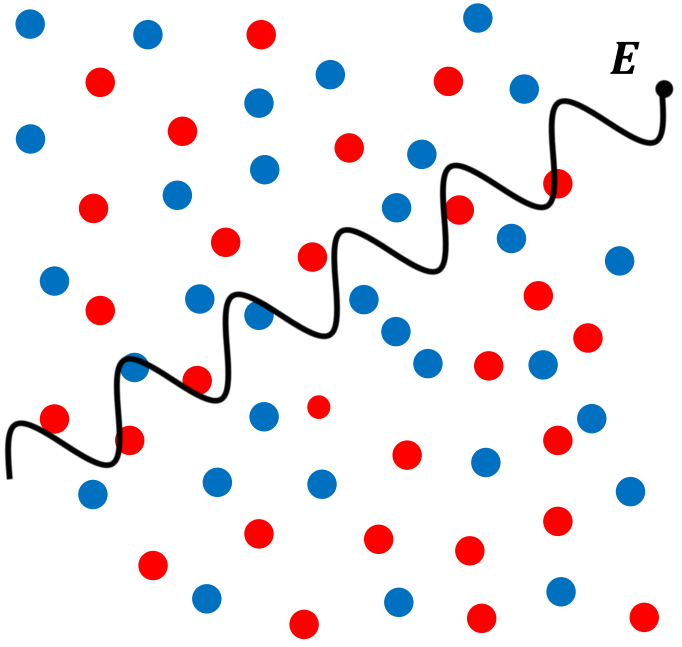


Illustration – applying external, time dependent EM field on positron-electron plasma

Boltzmann eq. for the distribution function $f(x,p)$ with **collision damping.**

$$p\cdot\partial f(x,p) + \underbrace{\boxed{qF^{\mu\nu}p_\nu\frac{\partial f(x,p)}{\partial p^\mu}}}_{\text{Vlasov force term}} = \underbrace{\boxed{C[f(x,p)]}}_{\text{Collisions}}$$

Collision term:

$$C[f(x,p)] = (p^\mu u_\mu)\kappa\left[f_{\text{eq}}(p)\frac{n(x)}{n_{\text{eq}}} - f(x,p)\right]$$

Formulation is manifestly covariant, gauge invariant and current conserving. It is also energy-momentum conserving when all plasma masses are equal. We can obtain the analytic solutions for the polarization tensor $\Pi^\mu_\nu(k)$ in the Fourier picture and the self-consistent solution for the EM 4-potential:

$$\tilde{j}^\mu_{\text{ind}}(k) = 2q\int(dp)p^\mu[\tilde{f}_+(k,p)-\tilde{f}_-(k,p)]$$

Induced current

$$\tilde{j}^\mu_{\text{ind}}(k) = \Pi^\mu_\nu(k)\tilde{A}^\nu(k)$$

Covariant Ohm's law

$$k_\mu\tilde{F}^{\mu\nu}(k) = \mu_0[\tilde{J}^\nu_{\text{free}}(k)+\tilde{j}^\nu_{\text{ind}}(k)]$$

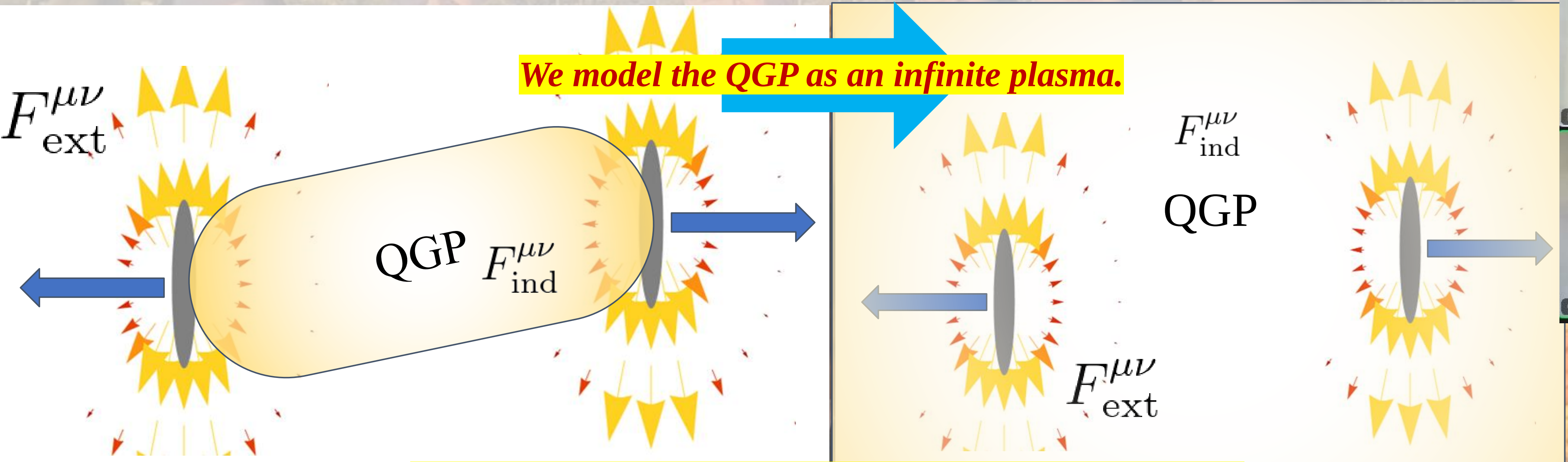
Maxwell's equations

M. Formanek, C. Grayson, J. Rafelski, & B. Müller. "Current-conserving relativistic linear response for collisional plasmas." Annals of Physics 434 (2021): 168605.

See Chris Grayson’s lecture:
Strong acceleration probed in heavy-ion collisions

Ultra-strong electromagnetic fields in heavy-ion collisions:

- Self-consistent combination of ion and QGP electromagnetic fields
- Probing dynamical EM pair production in QGP plasma



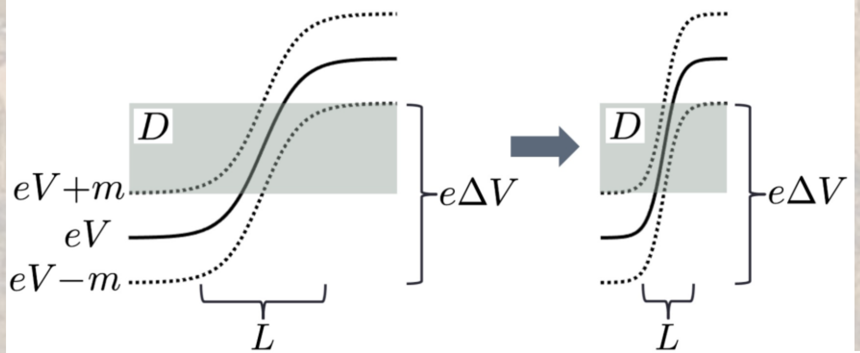
Qualitative illustrations of QGP formation and evolution

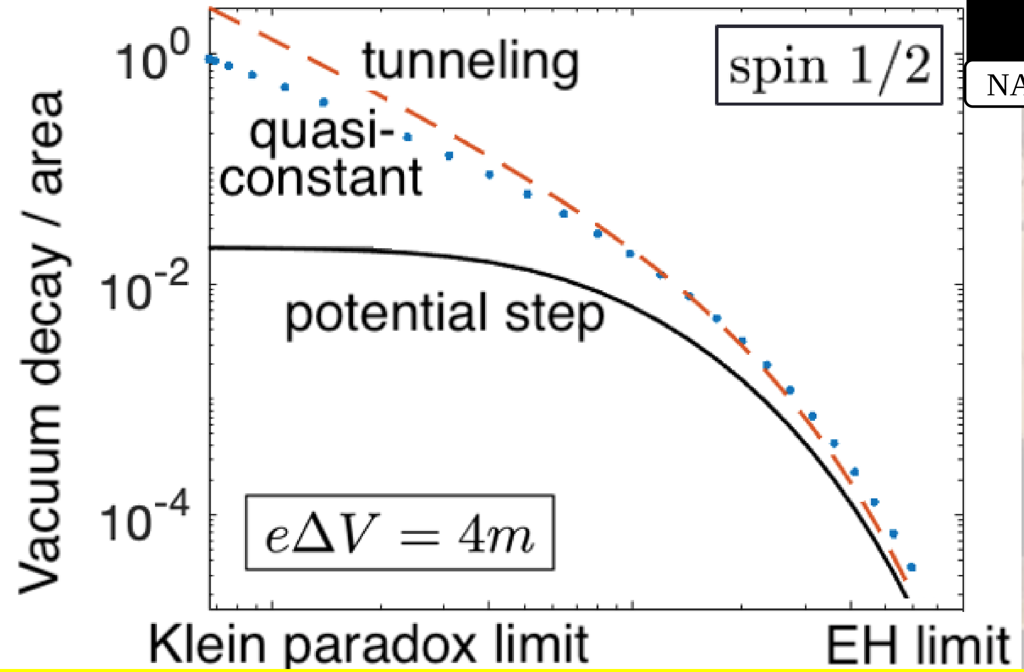
C. Grayson, M. Formanek, B. Müller, J. Rafelski, “Dynamic Magnetic Response of Quark-Gluon Plasma to Electromagnetic Fields” [arXiv:2204.14186] (2022)
M. Formanek, C. Grayson, J. Rafelski, B. Müller, “Current-Conserving Relativistic Linear Response for Collisional Plasmas” Annals of Physics 434 (2021)
K. Tuchin, “Particle production in strong electromagnetic fields in relativistic heavy-ion collisions.” Advances in High Energy Physics (2013)

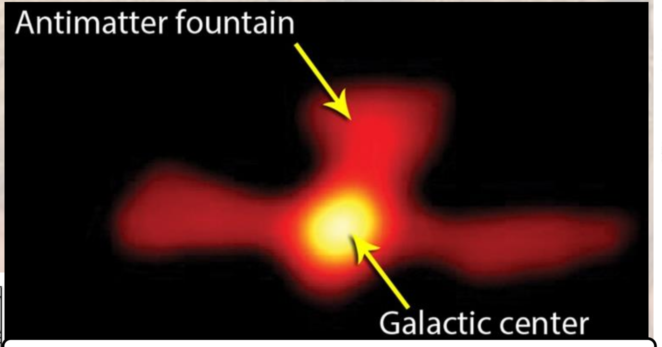
Sauter potential step

$$V_z = \frac{\mathcal{E}_0 L}{2}\tanh\left[\frac{2z}{L}\right]$$

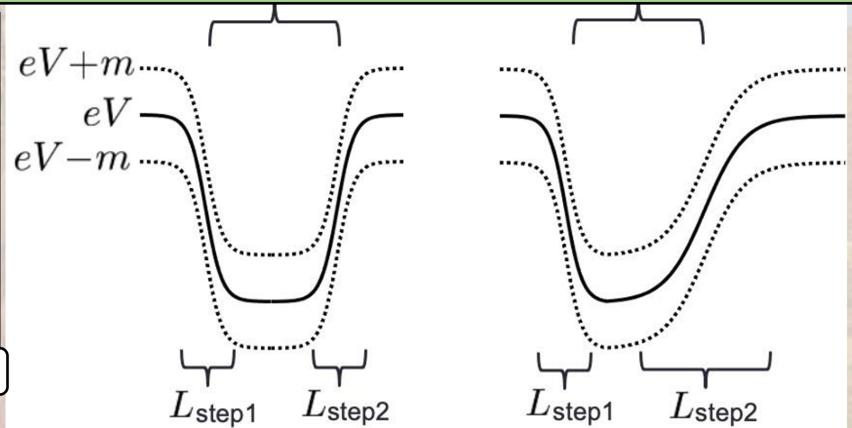
See Stefan Evan's lecture:
Surface pair production transition in pair production from Euler-Heisenberg to Klein paradox limit







NASA/W. Purcell, et al. APOD May 1st, 1997.



Step to well

Single potential step or two steps forming a well:
- Finite pair production per unit area versus the diverging rate per volume

Two steps forming a well required for:
- A good definition of vacuum
- Pair production highly sensitive to the shape of the well

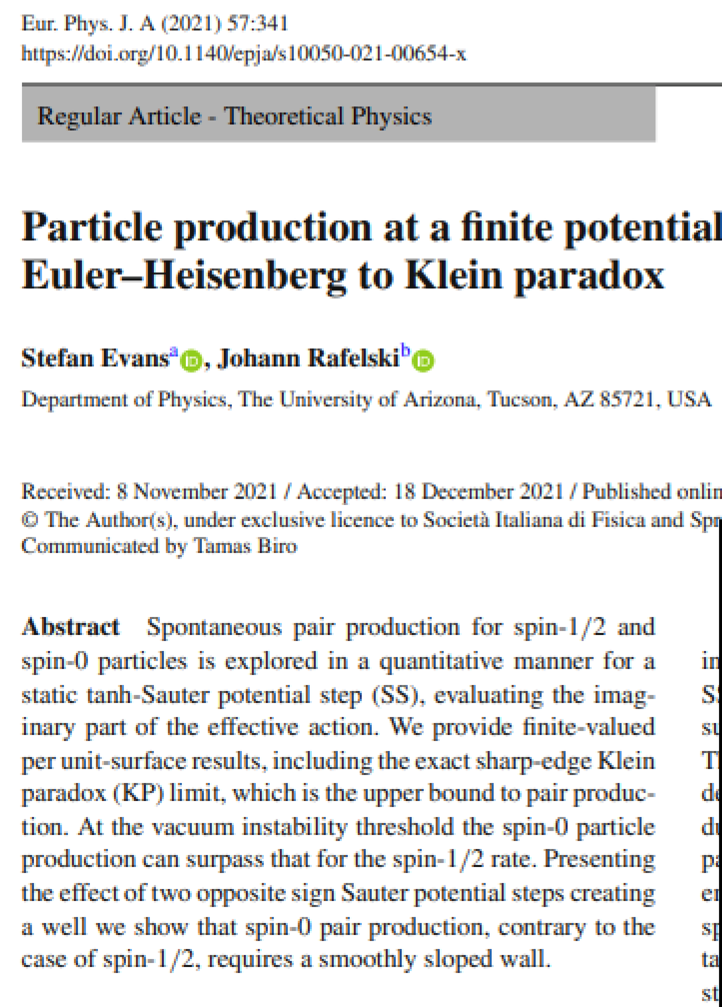
F. Sauter, "Zum 'Kleinschen Paradoxon'," Z. Phys. 73 (1932), 547-552 doi:10.1007/BF01349862

S. P. Kim, H. K. Lee and Y. Yoon, "Effective action of QED in electric field backgrounds. II. Spatially localized fields." Phys. Rev. D 82, 025015 (2010)

A. Chervyakov and H. Kleinert, "On Electron–Positron Pair Production by a Spatially Inhomogeneous Electric Field." Phys. Part. Nucl. 49 no.3, 374-396 (2018)

S. Evans and J. Rafelski. "Particle production at a finite potential step: Transition from Euler-Heisenberg to Klein paradox." (2021) [arXiv:2108.12959]

Amazing magnetic moment periodicity



Eur. Phys. J. A (2021) 57:341
https://doi.org/10.1140/epja/s10050-021-00654-x

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

Particle production at a finite potential step: transition from Euler–Heisenberg to Klein paradox

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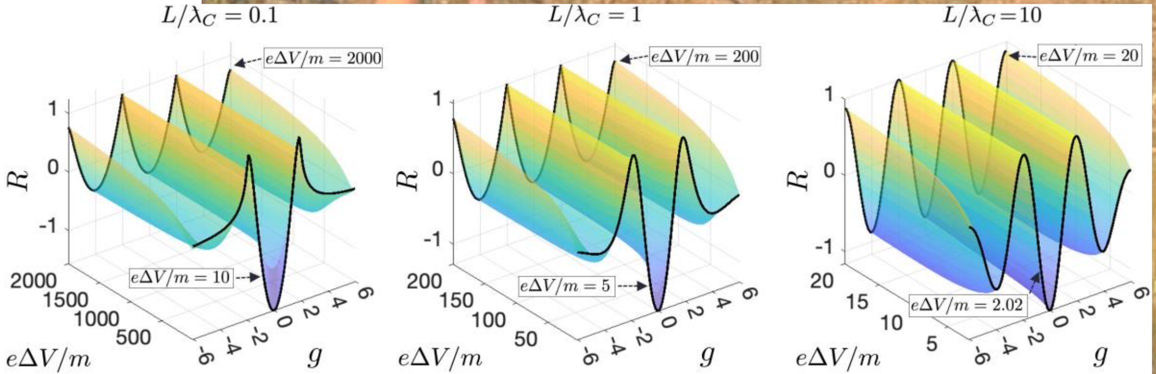
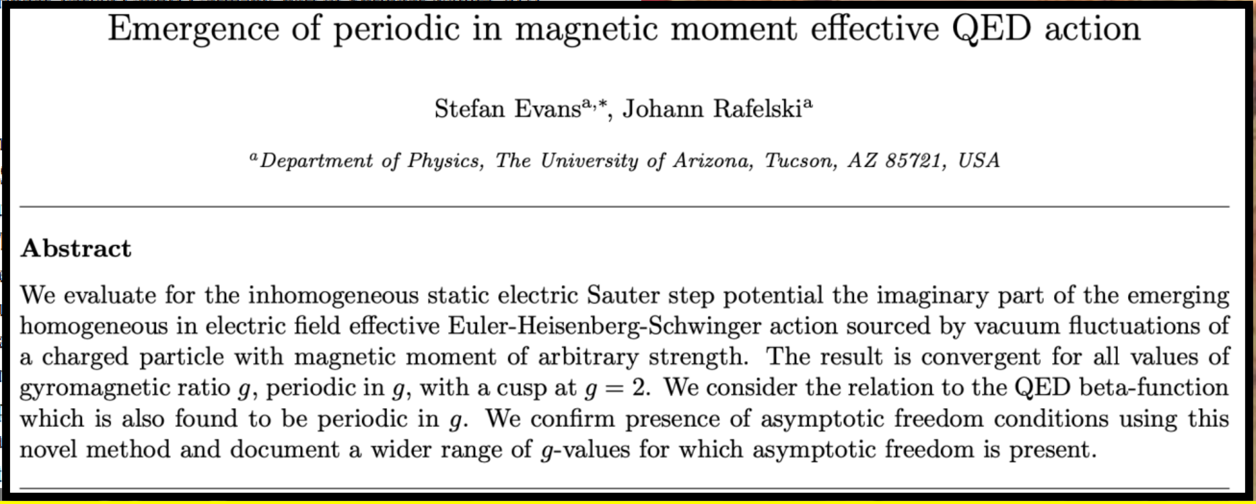
Received: 8 November 2021 / Accepted: 18 December 2021 / Published online: 31 December 2021

Communicated by Tamas Biro

Abstract Spontaneous pair production for spin-1/2 and spin-0 particles is explored in a quantitative manner for a static tanh-Sauter potential step (SS), evaluating the imaginary part of the effective action. We provide finite-valued per unit-surface results, including the exact sharp-edge Klein paradox (KP) limit, which is the upper bound to pair production. At the vacuum instability threshold the spin-0 particle production can surpass that for the spin-1/2 rate. Presenting the effect of two opposite sign Sauter potential steps creating a well we show that spin-0 pair production, contrary to the case of spin-1/2, requires a smoothly sloped wall.





Emergence of periodic in magnetic moment effective QED action

Stefan Evans[a,*], Johann Rafelski[a]

[a] *Department of Physics, The University of Arizona, Tucson, AZ 85721, USA*

Abstract

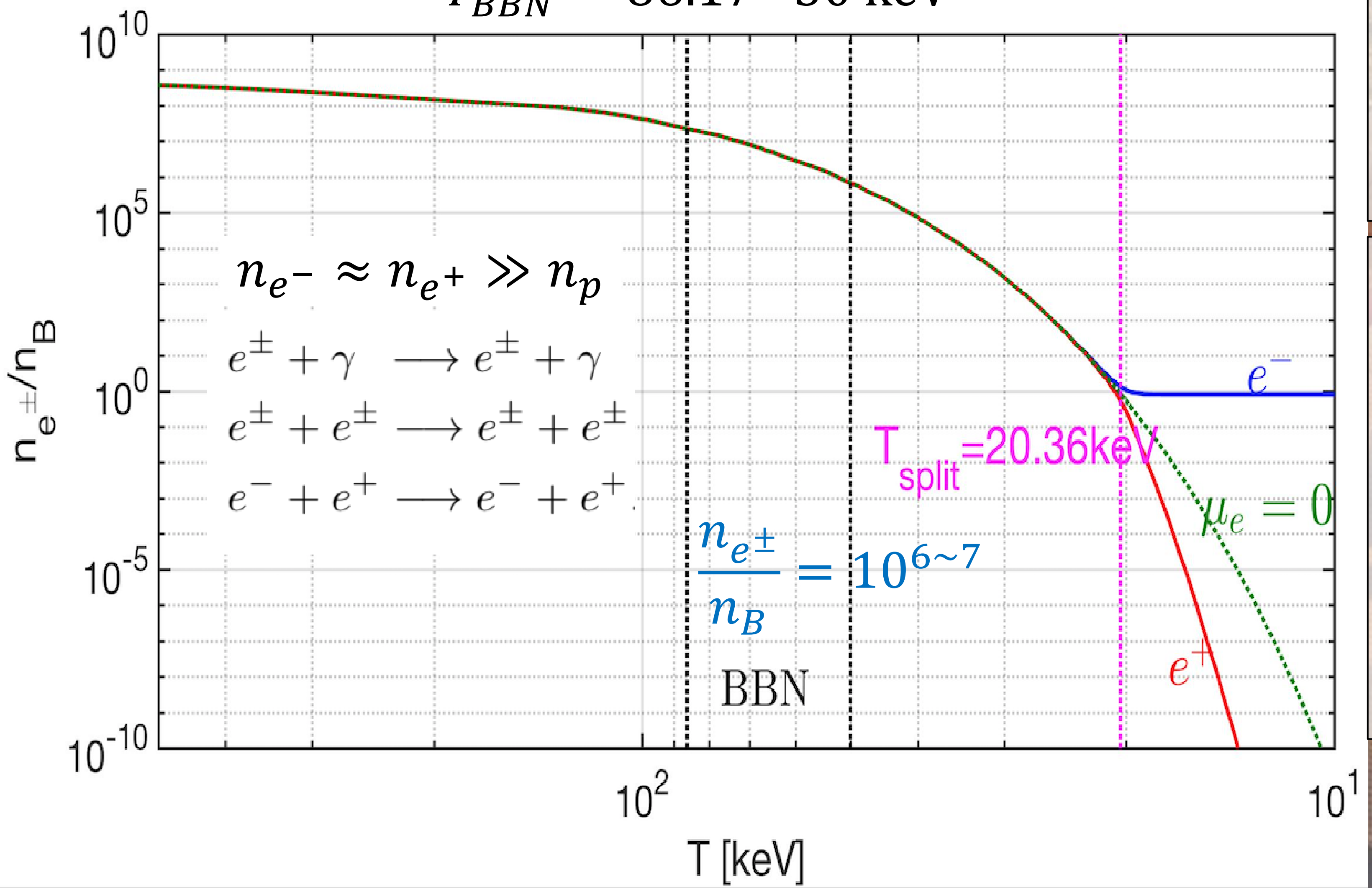
We evaluate for the inhomogeneous static electric Sauter step potential the imaginary part of the emerging homogeneous in electric field effective Euler-Heisenberg-Schwinger action sourced by vacuum fluctuations of a charged particle with magnetic moment of arbitrary strength. The result is convergent for all values of gyromagnetic ratio g, periodic in g, with a cusp at $g = 2$. We consider the relation to the QED beta-function which is also found to be periodic in g. We confirm presence of asymptotic freedom conditions using this novel method and document a wider range of g-values for which asymptotic freedom is present.

S. Evans and J. Rafelski. “Particle production at a finite potential step: Transition from Euler-Heisenberg to Klein paradox.” (2021) [arXiv:2108.12959]
S. Evans, and J. Rafelski. "Emergence of periodic in magnetic moment effective QED action." arXiv preprint arXiv:2203.13145 (2022).

See Cheng-Tao Yang's lecture:

How does the dense electron/positron plasma affect Big Bang nucleosynthesis (BBN) reactions?

$T_{BBN} = 86.17 \sim 50$ keV



$$n_{e^-} \approx n_{e^+} \gg n_p$$

$$e^\pm + \gamma \longrightarrow e^\pm + \gamma$$
$$e^\pm + e^\pm \longrightarrow e^\pm + e^\pm$$
$$e^- + e^+ \longrightarrow e^- + e^+$$

$$\frac{n_{e^\pm}}{n_B} = 10^{6\sim7}$$

The presence of electron/positron during BBN could play an important role in the formation of the light elements in BBN.

We use linear response theory **(See M. Formanek's lecture)** adapted by **C. Grayson** to describe the inter-nuclear potential of e^+e^- plasma.

We improve upon prior efforts by evaluation and inclusion of the **collision damping rate *κ*** due to scattering in the dense plasma medium.

Part IV:
Short report on more exciting work from the group

Will Price

Completing EM interactions: Covariant classical radiation reaction

$$\tau_0 = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 mc^3}$$

$$P^{\mu\nu} = g^{\mu\nu} - \frac{u^\mu u^\nu}{u^2}$$

Principle models:

$$ma^\mu = \frac{e}{c}F^{\mu\nu}u_\nu + m\tau_0\left(\frac{da^\mu}{d\tau} + \frac{a_\nu a^\nu}{c^2}u^\mu\right)$$

Lorentz-Abraham-Dirac (LAD) ←

As far as Jackson text goes

$$ma^\mu = \frac{e}{c}F^{\mu\nu}u_\nu + e\tau_0\left(u\cdot\partial F^{\mu\nu}u_\nu + \frac{e}{m}P^{\mu\nu}F_{\nu\alpha}F^{\alpha\beta}u_\beta\right)$$

Landau-Lifshitz (LL) ←

As far as LL text goes

$$ma^\mu = \frac{e}{c}F^{\mu\nu}u_\nu + \tau_0 P^\mu_\nu \frac{d}{d\tau}\left(\frac{e}{c}F^{\nu\alpha}u_\alpha\right)$$

Eliezer-Ford-O’Connell (EFO) ←

The Cinderella of RR?

W. Price, M. Formanek, and J. Rafelski. “Radiation reaction and limiting acceleration”. PRD 105 (2022)

P. A. M. Dirac, “Classical theory of radiating electrons,” Proc. R. Soc. A 167, 148 (1938)

L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, 2ed, London, England: Pergamon (1962)

S. E. Gralla, A. I. Harte, R. M. Wald. “A Rigorous Derivation of Electromagnetic Self-force.” Rev. D80, 024031(2009)

Distinct features of radiation reaction models

LAD

- Requires self-interaction
- Unphysical runaway solutions
- Computationally impossible

Kinematic variables only
$\boldsymbol{a^\mu, \dot{a}^\mu}$

LL

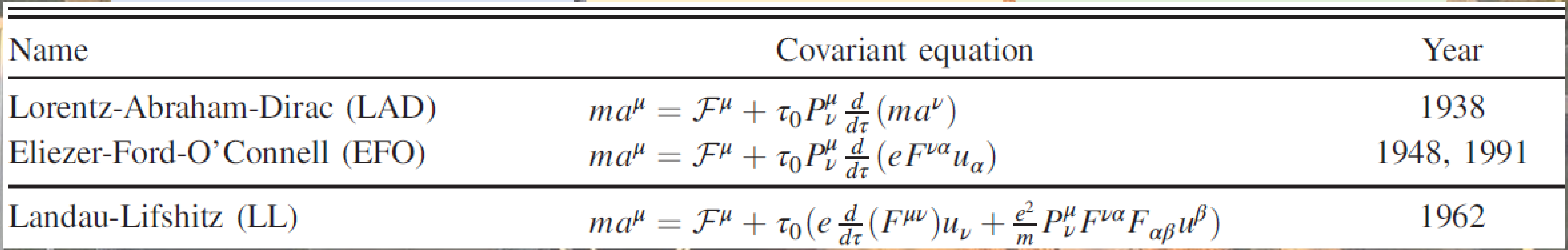
- Equivalent to LAD in perturbative limit
- Useless for strong accelerations

Field variables only
$\boldsymbol{F^{\mu\nu}, \dot{F}^{\mu\nu}}$

EFO

- Maximum limiting acceleration.
- Equivalent to LL for weak acceleration.

Kinematic and Fields
$\boldsymbol{a^\mu, \dot{F}^{\mu\nu}}$



Name	Covariant equation	Year
Lorentz-Abraham-Dirac (LAD)	$ma^\mu = \mathcal{F}^\mu + \tau_0 P^\mu_\nu \frac{d}{d\tau}(ma^\nu)$	1938
Eliezer-Ford-O'Connell (EFO)	$ma^\mu = \mathcal{F}^\mu + \tau_0 P^\mu_\nu \frac{d}{d\tau}(eF^{\nu\alpha}u_\alpha)$	1948, 1991
Landau-Lifshitz (LL)	$ma^\mu = \mathcal{F}^\mu + \tau_0(e\frac{d}{d\tau}(F^{\mu\nu})u_\nu + \frac{e^2}{m}P^\mu_\nu F^{\nu\alpha}F_{\alpha\beta}u^\beta)$	1962

W. Price, M. Formanek, and J. Rafelski. “Radiation reaction and limiting acceleration”. PRD 105 (2022)
P. A. M. Dirac, “Classical theory of radiating electrons,” Proc. R. Soc. A 167, 148 (1938)
L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, 2ed, London, England: Pergamon (1962)
S. E. Gralla, A. I. Harte, R. M. Wald. “A Rigorous Derivation of Electromagnetic Self-force.” Rev. D80, 024031(2009)

Radiation reaction and limiting acceleration

$$\tau_0 = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 m_e c^3} = 6.27\times 10^{-24}\ \mathrm{s}$$

Eliezer-Ford-O'Connell (EFO) in homogenous fields

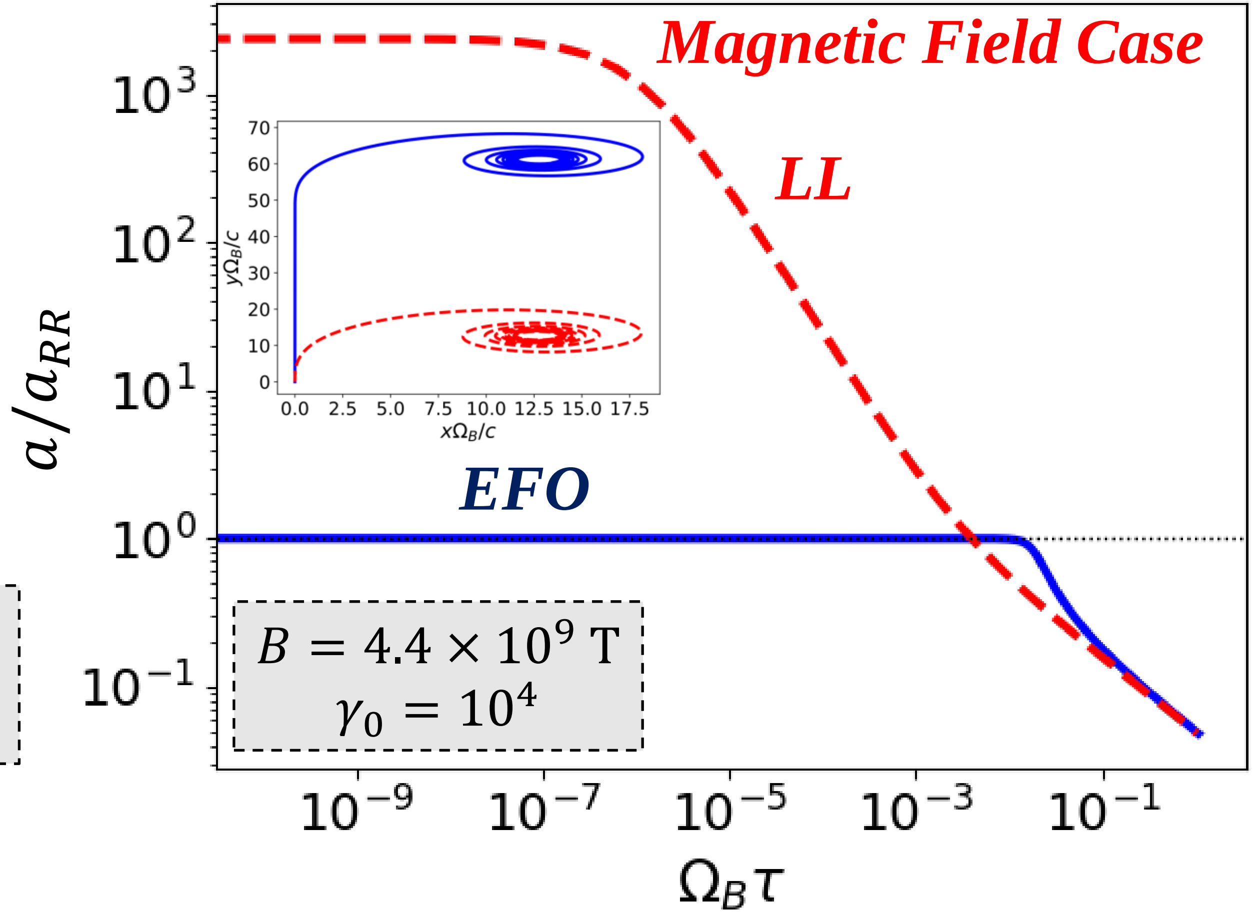
$$a^2 = -a_{LF}^2\,\frac{1+\tau_0^2\,\dfrac{e^4}{m^4}\dfrac{c^2\mathcal{P}^2}{|a_{LF}^2|}}{1+\tau_0^2\left(\dfrac{e^2}{m^2}2\mathcal{S}+\dfrac{|a_{LF}^2|}{c^2}\right)}$$

$$\lim_{\gamma\to\infty} a^2 \to -\frac{c^2}{\tau_0^2}$$

$$\xrightarrow{}$$

$$|a_{RR}| = \frac{c}{\tau_0} = \frac{3}{2\alpha}a_{cr}$$

Limiting acceleration: A common feature with Born-Infeld EM theory



W. Price, M. Formanek, and J. Rafelski. "Radiation reaction and limiting acceleration". PRD 105 (2022)

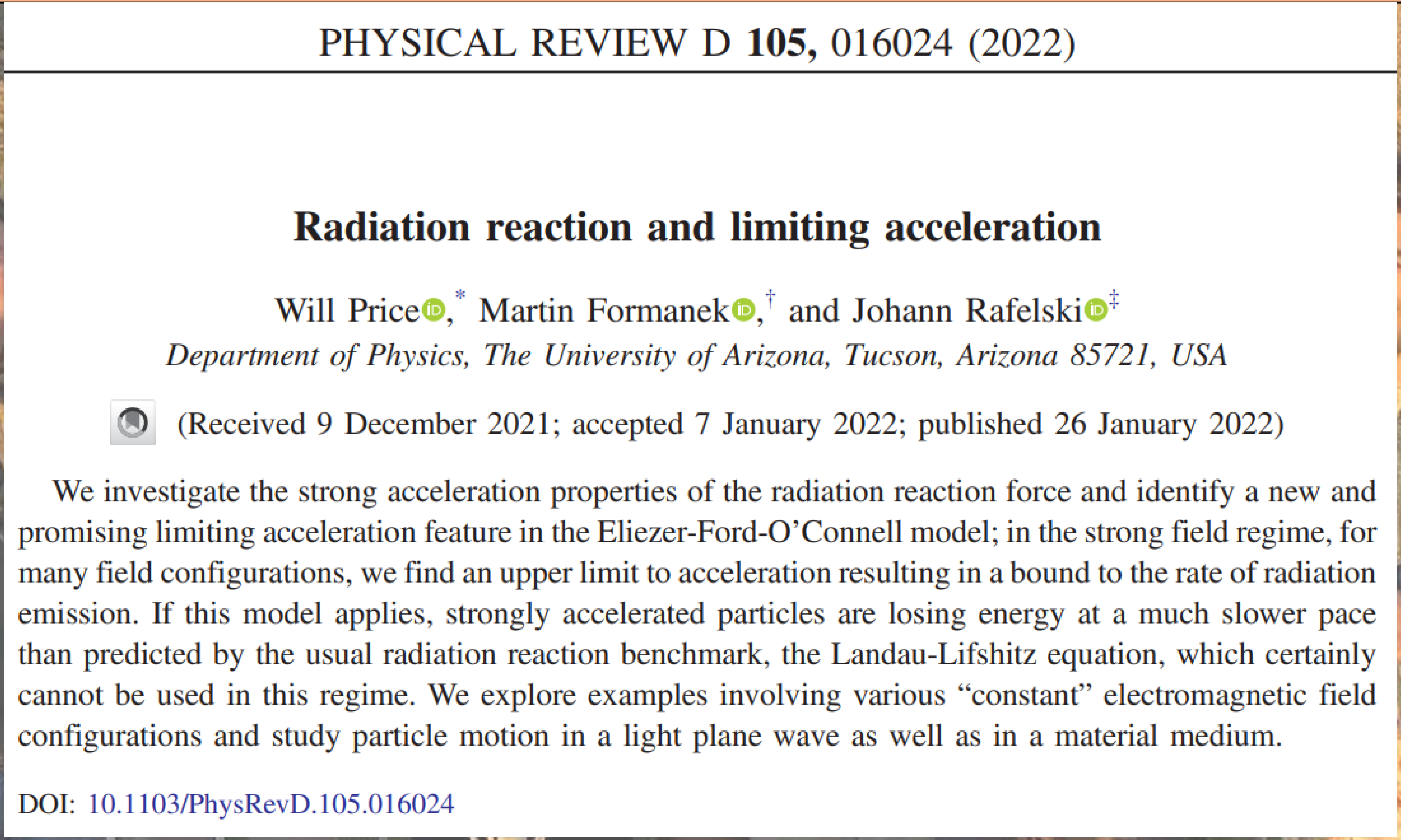
M. Born and L. Infeld. "Foundations of the new field theory." Proc. Roy. Soc. Lond. A 144, no.852, 425 (1934)

I. Birula. "Nonlinear Electrodynamics: Variations On A Theme By Born And Infeld." In: B. Jancewicz, J. Lukierski: Quantum Theory Of Particles and Fields, World Scientific (1983)

$$a_{LF}^{\mu} = \frac{e}{m}F^{\mu\nu}u_\nu$$

$$\Omega_B = \frac{eB}{m}$$

As we just saw,
new research in SR happens every day!



PHYSICAL REVIEW D **105,** 016024 (2022)

Radiation reaction and limiting acceleration

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Department of Physics, The University of Arizona, Tucson, Arizona 85721, USA

(Received 9 December 2021; accepted 7 January 2022; published 26 January 2022)

We investigate the strong acceleration properties of the radiation reaction force and identify a new and promising limiting acceleration feature in the Eliezer-Ford-O’Connell model; in the strong field regime, for many field configurations, we find an upper limit to acceleration resulting in a bound to the rate of radiation emission. If this model applies, strongly accelerated particles are losing energy at a much slower pace than predicted by the usual radiation reaction benchmark, the Landau-Lifshitz equation, which certainly cannot be used in this regime. We explore examples involving various “constant” electromagnetic field configurations and study particle motion in a light plane wave as well as in a material medium.

DOI: 10.1103/PhysRevD.105.016024

W. Price, M. Formanek, and J. Rafelski. “Radiation reaction and limiting acceleration”. PRD 105 (2022)

Andrew Steinmetz

Path warping (2020)
The new idea for radiation reaction

Omitting problematic "Schott term"

$$m\tau_0\ddot{u}^\mu$$

Start with point external force + Larmor term

$$m\dot{u}^\mu = f^\mu + m\tau_0\frac{\dot{u}^2}{c^2}u^\mu$$

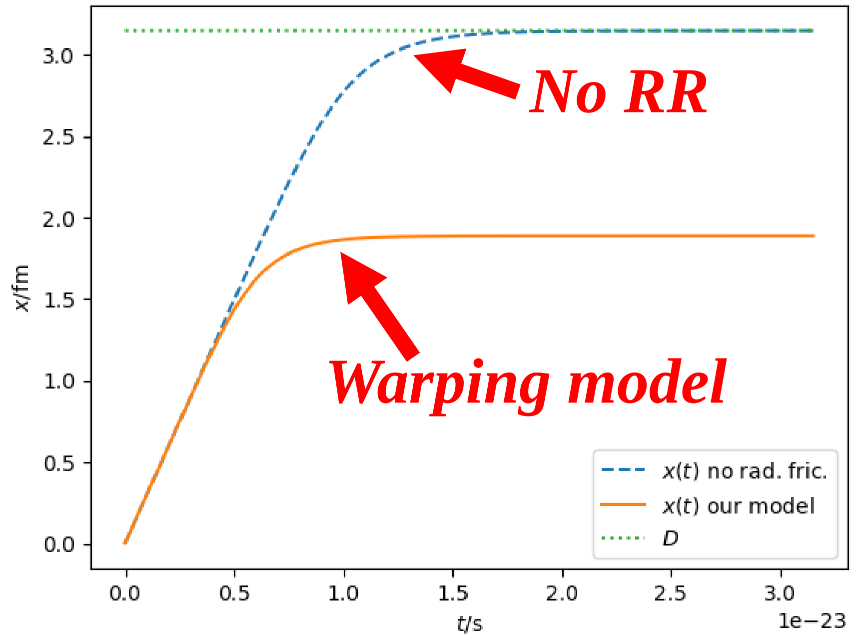
Introduce "path warping" for particles with medium friction

$$u\cdot\dot{u} = \tau_0\dot{u}^2 \neq 0 \quad \longleftrightarrow \quad u^2 = w_{\mu\nu}u^\mu u^\nu = c^2$$

Path warping along world-line of particle

$$u\cdot\dot{u} = -\frac{1}{2}\frac{dw_{\mu\nu}}{d\tau}u^\mu u^\nu \neq 0$$

Doubled particle stopping power.
Applications for quark jet quenching in QGP



Outlook: We hope to develop an action principle accounting for RR by warping in vacuum.

M. Formanek, A. Steinmetz, and J. Rafelski. "Radiation reaction friction: Resistive material medium." Physical Review D 102.5 (2020): 056015.

What about acceleration by torque due to magnetic force?

The Thomas-Bargmann-Michel-Telegdi (TBMT) equation is

$$\frac{ds^\alpha}{d\tau} = \frac{e}{m}\left[\frac{g}{2}F^{\alpha\beta}s_\beta + \left(\frac{g}{2} - 1\right)u^\alpha\left(s_\alpha F^{\alpha\beta}u_\beta\right)\right]$$

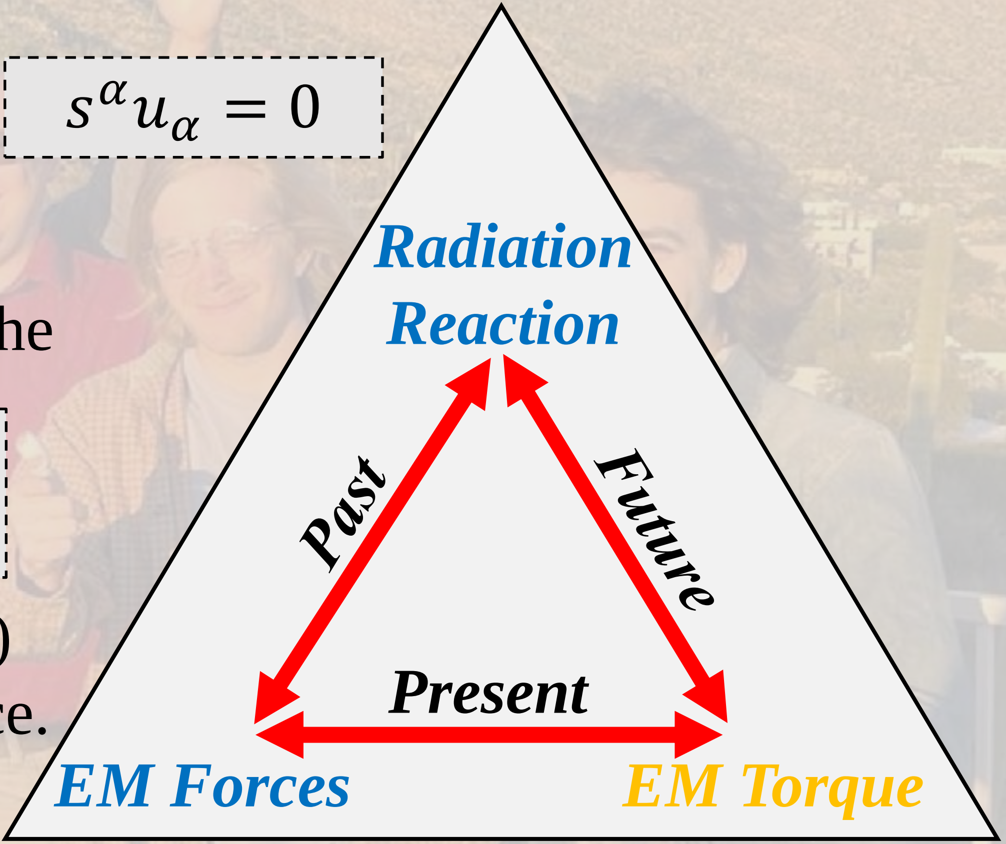
$$s^\alpha u_\alpha = 0$$

This describes the torque experienced by a particle with spin under the influence of homogenous EM fields via the Lorentz force.

$$\frac{du^\alpha}{d\tau} = \frac{e}{m}F^{\alpha\beta}u_\beta$$

This alone does not describe SGF.

a) Inhomogeneous fields where the Stern-Gerlach (SG) magnetic dipole force must be added to Lorentz force.
b) Interplay between dipole forces and torque.
c) Interplay between dipole forces and radiation reaction.



V. Bargmann, L. Michel, and V. L. Telegdi, Phys. Rev. Lett. 2, 435 (1959).
J. Rafelski, M. Formanek, and A. Steinmetz. "Relativistic dynamics of point magnetic moment." EPJC 78.1 (2018): 1-12.
J. Schwinger. "Spin precession - a dynamical discussion." American Journal of Physics 42.6 (1974): 510-513.

More need to complete EM interactions: Unified covariant classical magnetic dipole interaction

Electric energy: $E_{el} = ecA^0$

Magnetic energy: $E_{mag} = d_m c B^0$

Magnetic dipole charge

$$\boldsymbol{\mu} = (d_m c)\boldsymbol{S}$$

A covariant magnetic potential B^μ can be introduced

$$B_\mu \equiv F^*_{\mu\nu} s^\nu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} s^\nu$$

Define a Force Field Tensor

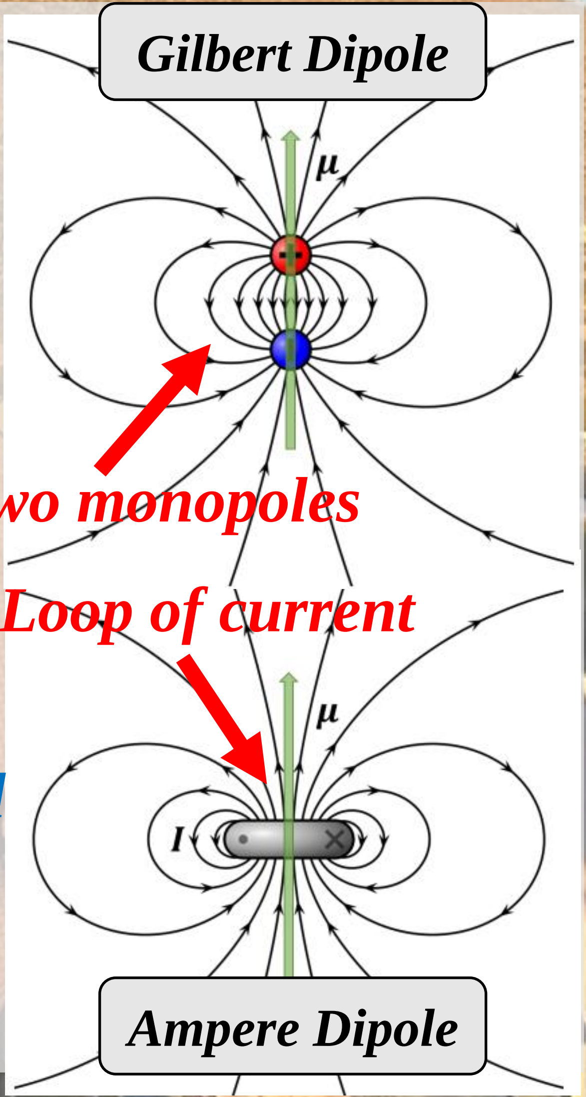
$$G^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu$$

Point particle classical Lagrangian

$$L = mc\sqrt{u^2} + eA \cdot u + d_m B \cdot u$$

Covariant description contains both Gilbert and Ampere dipole structure

This formulation incorporates the magnetic moment d_m as an elementary property of particles like charge and mass.



Two monopoles

Loop of current

J. Rafelski, M. Formanek, and A. Steinmetz. "Relativistic dynamics of point magnetic moment." EPJC 78.1 (2018): 1-12.

Completing EM interactions: Unified covariant classical magnetic dipole interaction

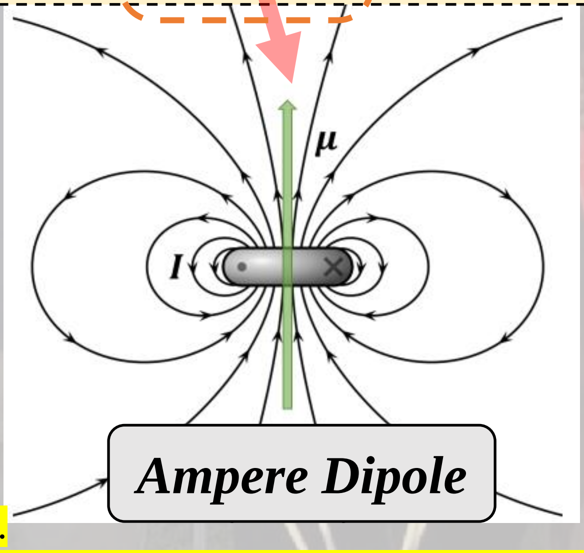
The equations of motion for the above are then

$$\dot{u}^{\mu} = \frac{e}{m} F^{\mu\nu} u_{\nu} - \frac{d_m}{m} s \cdot \partial (F^{*\mu\nu}) u_{\nu} - \frac{d_m}{m} \mu_0 \epsilon^{\gamma\alpha\beta\mu} j_{\gamma} u_{\alpha} s_{\beta}$$

Amazingly, Martin can solve this complex equation exactly for several examples.

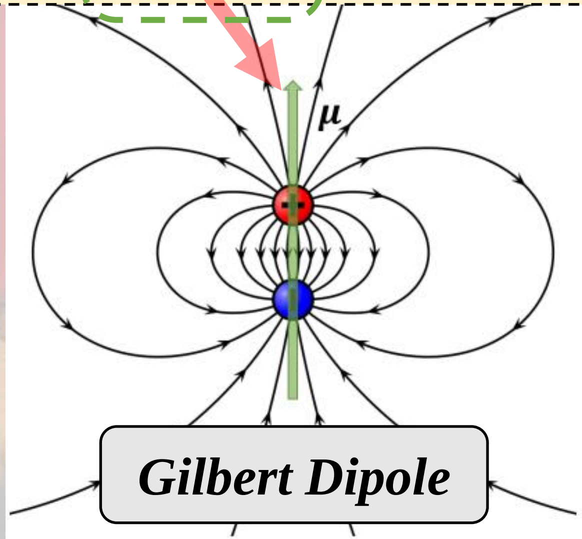
Comoving Frame (CF)

$$\boldsymbol{F}\Big|_{\mathrm{CF}} = e\boldsymbol{E} + \boxed{\boldsymbol{\nabla}(\boldsymbol{\mu}\cdot\boldsymbol{B})} - \boldsymbol{\mu}\times\frac{\partial \boldsymbol{E}}{\partial t} = e\boldsymbol{E} + \boxed{(\boldsymbol{\mu}\cdot\boldsymbol{\nabla})\boldsymbol{B}} + \mu_0 \boldsymbol{\mu}\times\boldsymbol{j}$$



Ampere Dipole

OR



Gilbert Dipole

M. Formanek, A. Steinmetz, and J. Rafelski.
“Motion of classical charged particles with magnetic moment in external plane-wave electromagnetic fields.” Physical Review A 103.5 (2021): 052218.
J. Rafelski, M. Formanek, and A. Steinmetz. "Relativistic dynamics of point magnetic moment." EPJC 78.1 (2018): 1-12.

Quantum magnetic dipoles:
Diverse forms of quantum equations

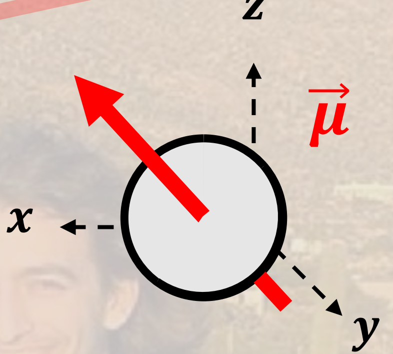
$$\boldsymbol{\mu} \leftarrow \frac{g}{2}\frac{e\hbar}{2m}$$

$$\frac{g}{2} = 1 + a$$

Non-relativistic magnetic dipole has the Hamiltonian:

$$\widehat{H}_{Mag.} = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}}$$

Relativistic magnetic dipoles have a diversity of models:



$$\left(\gamma\cdot(i\hbar\partial - eA) - mc - \left(\boldsymbol{\mu} - \frac{e\hbar}{2m}\right)\frac{1}{2c}\sigma_{\alpha\beta}F^{\alpha\beta}\right)\psi = 0$$

Dirac-Pauli (DP)

$$\left((i\hbar\partial - eA)^2 - m^2c^2 - \boldsymbol{\mu} m\sigma_{\alpha\beta}F^{\alpha\beta}\right)\psi = 0$$

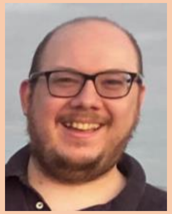
Klein-Gordon-Pauli (KGP)

$$\left((i\hbar\partial - eA)^2 - \widetilde{m}^2c^2\right)\psi = 0$$

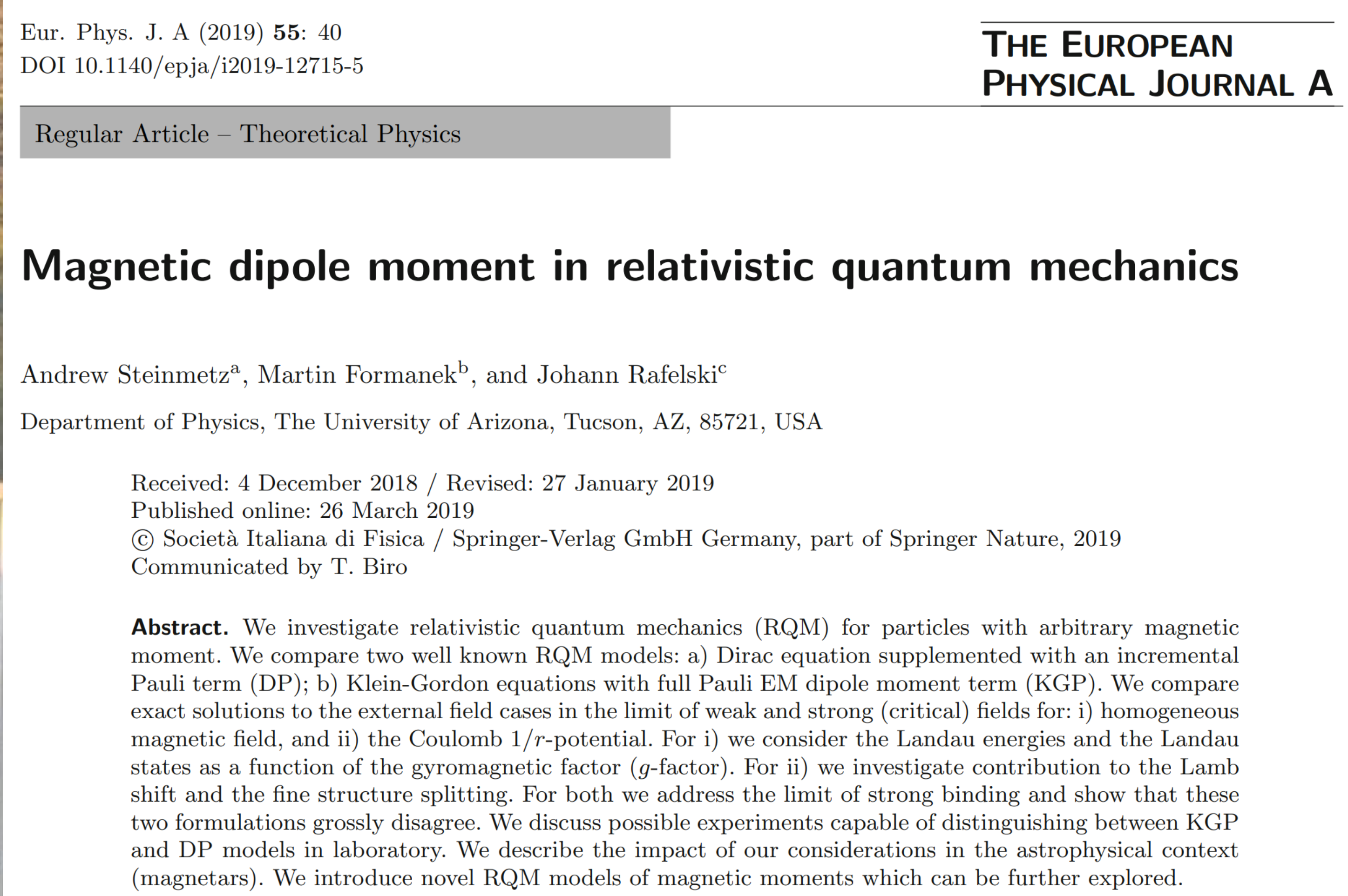
"Improved" Klein-Gordon-Pauli (IKGP)

$$\widetilde{m}c = mc + \boldsymbol{\mu}\frac{1}{2c}\sigma_{\alpha\beta}F^{\alpha\beta} \longrightarrow \widetilde{m}^2c^2 = m^2c^2 + \boldsymbol{\mu} m\sigma_{\alpha\beta}F^{\alpha\beta} + \boldsymbol{\mu}^2\frac{1}{4c^2}\left(\sigma_{\alpha\beta}F^{\alpha\beta}\right)^2$$

A. Steinmetz, M. Formanek, and J. Rafelski. "Magnetic dipole moment in relativistic quantum mechanics." EPJA 55.3 (2019): 1-17.
R. P. Feynman, and M. Gell-Mann. "Theory of the Fermi interaction." Physical Review 109.1 (1958)
M. Veltman, "Two component theory and electron magnetic moment," Acta Phys. Polon. B 29 (1998) 783 [hep-th/9712216]



Strong Coulomb field eigen-energies



Eur. Phys. J. A (2019) **55**: 40
DOI 10.1140/epja/i2019-12715-5

THE EUROPEAN
PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Magnetic dipole moment in relativistic quantum mechanics

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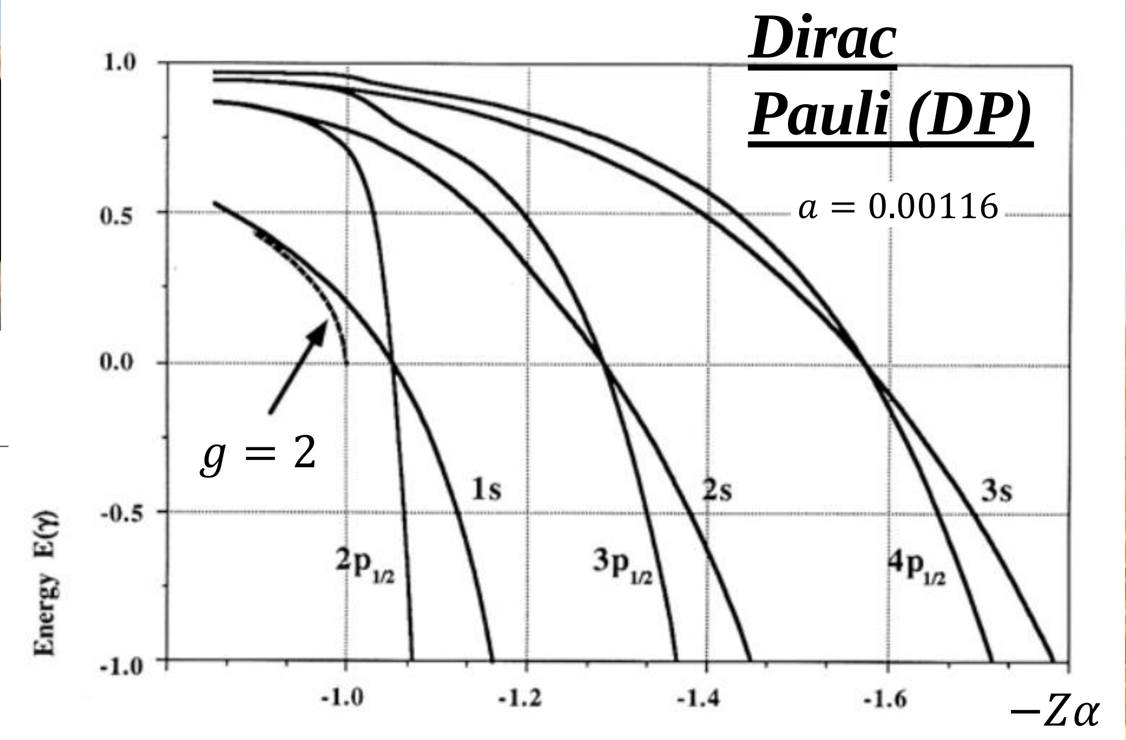
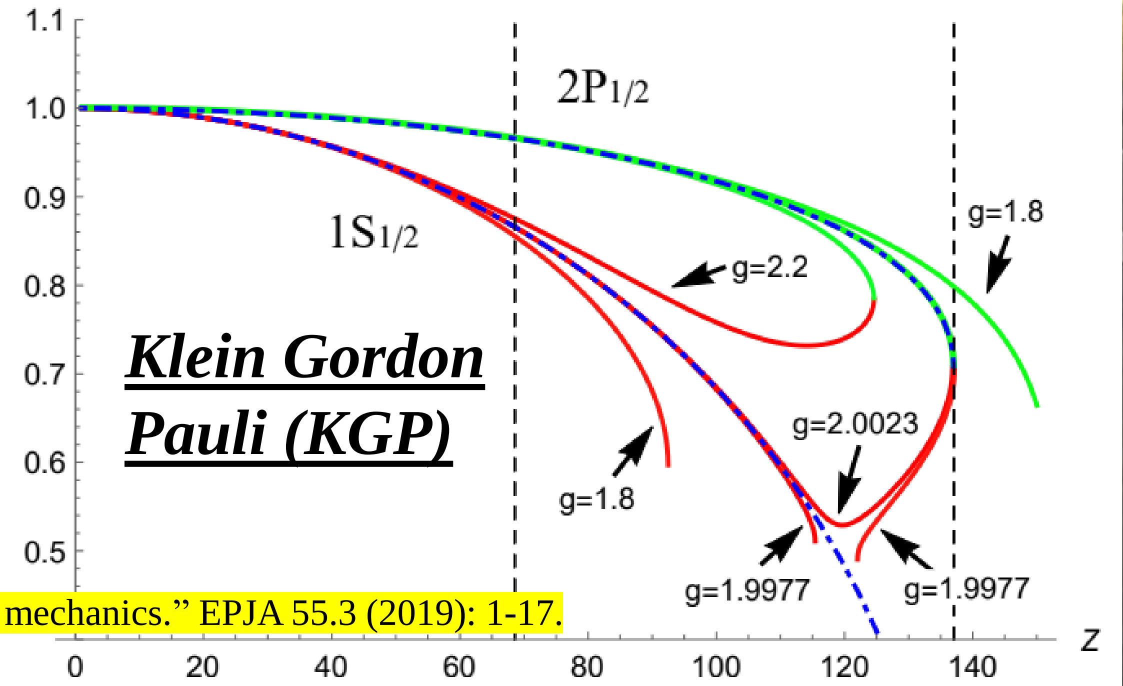
Received: 4 December 2018 / Revised: 27 January 2019

Communicated by T. Biro

Abstract. We investigate relativistic quantum mechanics (RQM) for particles with arbitrary magnetic moment. We compare two well known RQM models: a) Dirac equation supplemented with an incremental Pauli term (DP); b) Klein-Gordon equations with full Pauli EM dipole moment term (KGP). We compare exact solutions to the external field cases in the limit of weak and strong (critical) fields for: i) homogeneous magnetic field, and ii) the Coulomb $1/r$-potential. For i) we consider the Landau energies and the Landau states as a function of the gyromagnetic factor (g-factor). For ii) we investigate contribution to the Lamb shift and the fine structure splitting. For both we address the limit of strong binding and show that these two formulations grossly disagree. We discuss possible experiments capable of distinguishing between KGP and DP models in laboratory. We describe the impact of our considerations in the astrophysical context (magnetars). We introduce novel RQM models of magnetic moments which can be further explored.





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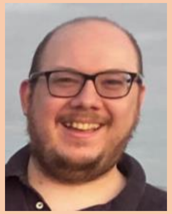
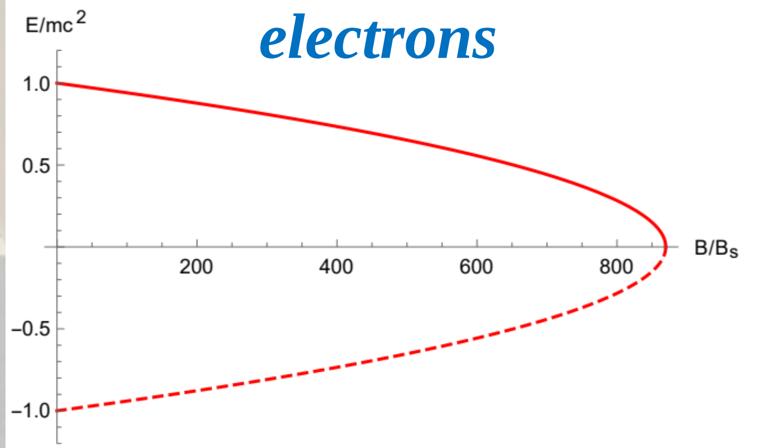


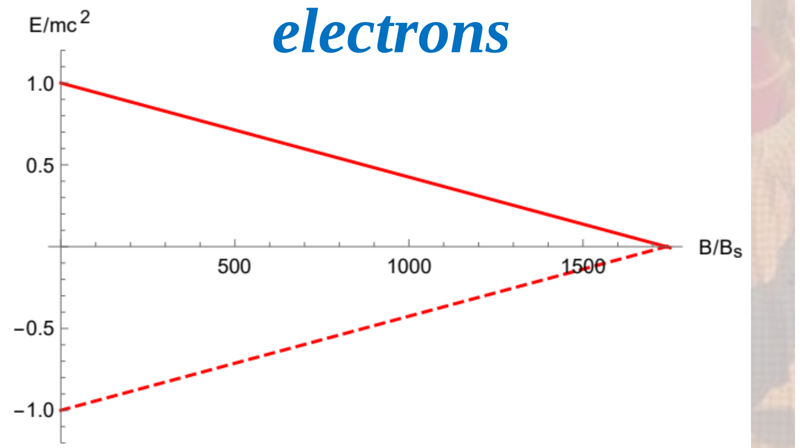
Constant magnetic field eigen-energies

$$B_S \equiv \frac{m^2c^2}{e\hbar} = \begin{cases} 4.41 \times 10^9\ \mathrm{T\ (electrons)} \\ 1.49 \times 10^{16}\ \mathrm{T\ (protons)} \end{cases}$$

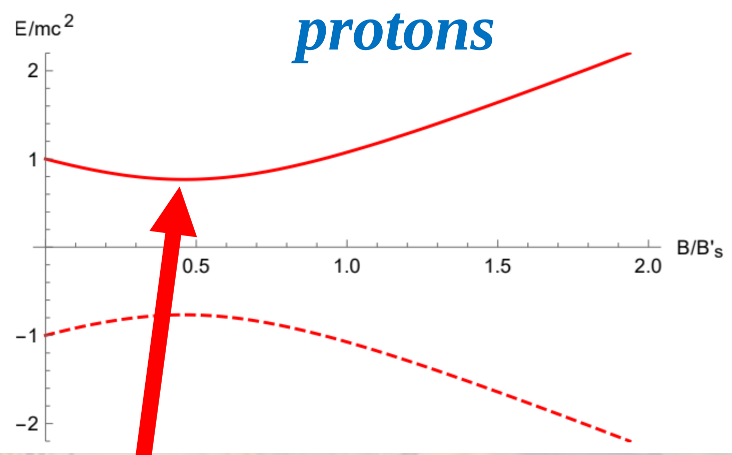
Klein-Gordon-Pauli (KGP)



Dirac-Pauli (DP)



"Improved" Klein-Gordon-Pauli (IKGP)



$$B_{min} \equiv \frac{mc^2}{\mu}\left(\mu - \frac{e\hbar}{2m}\right)$$

Expect grossly different (KGP vs DP vs IKGP) magnetization properties in magnetars.

A. Steinmetz, M. Formanek, and J. Rafelski. "Magnetic dipole moment in relativistic quantum mechanics." EPJA 55.3 (2019): 1-17.
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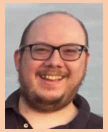


Andrew Steinmetz's work in preparation

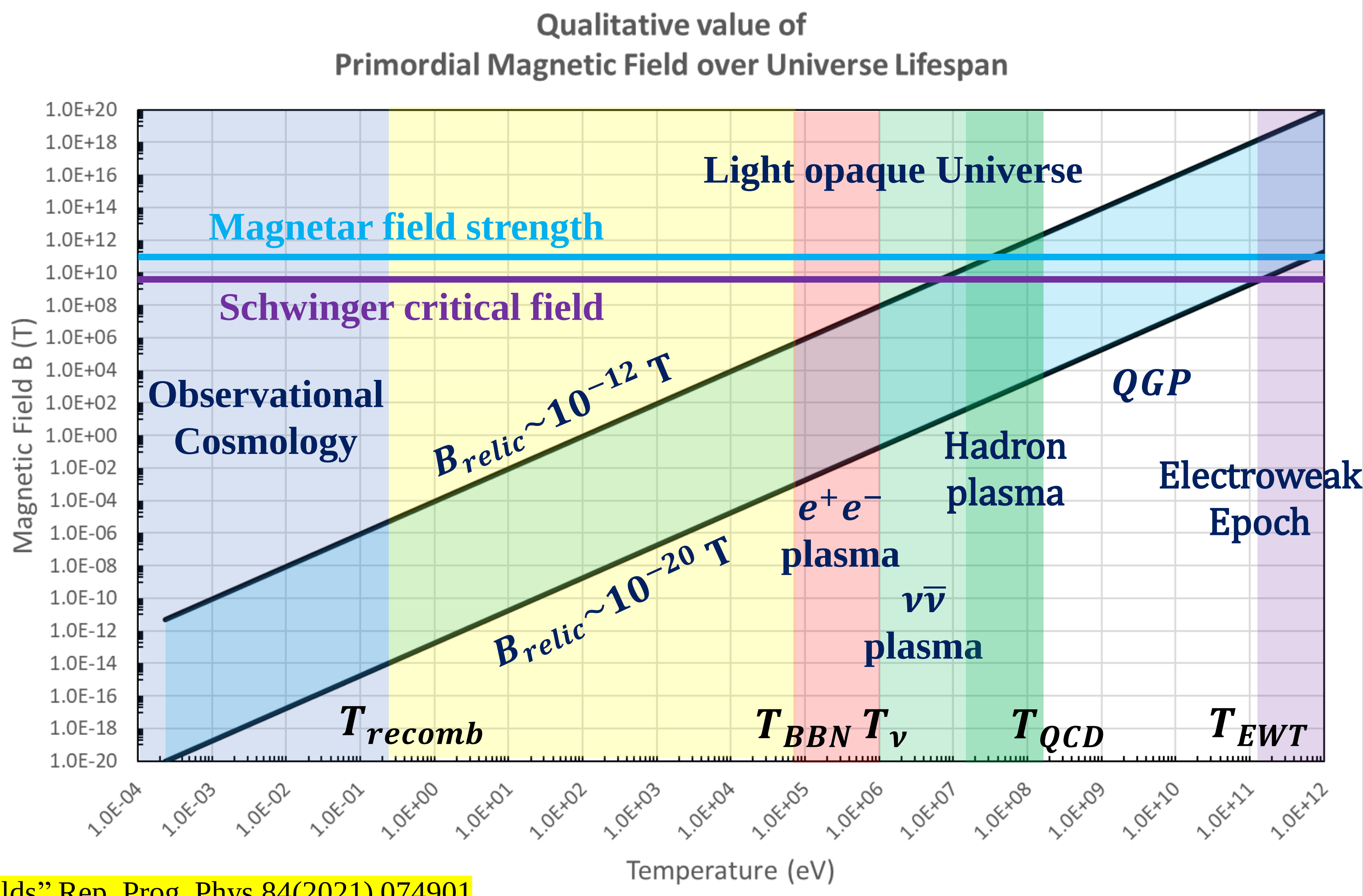
The intergalactic magnetic field B_{relic} is not strongly constrained at the megaparsec scale:

$$10^{-20}\ \mathrm{T} < B_{relic} < 5 \times 10^{-12}\ \mathrm{T}$$

The relic fields likely diluted in the universe's expansion due to the conservation of magnetic flux $\boldsymbol{M}(t) \to 0$

$$\boldsymbol{B}(t) = \boldsymbol{M}(t) + \frac{\boldsymbol{B}_{relic}}{a(t)^2}$$

We expect CP violation to depend on transition magnetic moments in the presence of magnetic fields.



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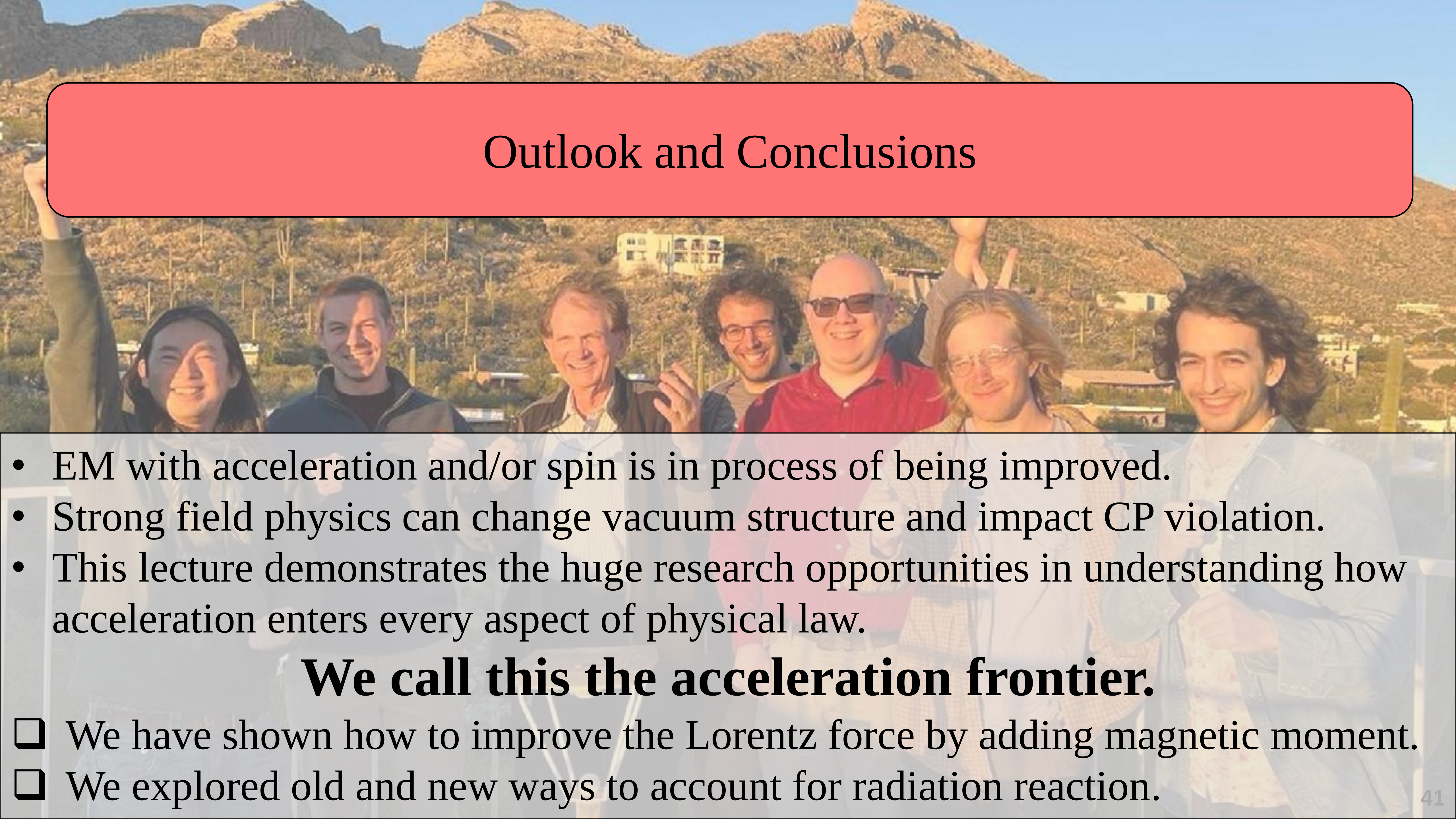


Outlook and Conclusions

- EM with acceleration and/or spin is in process of being improved.
- Strong field physics can change vacuum structure and impact CP violation.
- This lecture demonstrates the huge research opportunities in understanding how acceleration enters every aspect of physical law.

We call this the acceleration frontier.

- ☐ We have shown how to improve the Lorentz force by adding magnetic moment.
- ☐ We explored old and new ways to account for radiation reaction.

Supplementary Slides

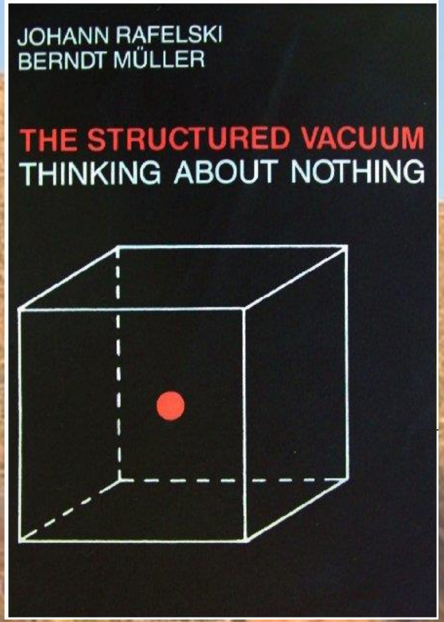
After fourteen years (1919/1920) Einstein brings back the Aether

"It would have been more correct if I had limited myself, in my earlier publications, to emphasizing only the non-existence of an æther velocity, instead of arguing the total non-existence of the æther, for I can see that with the word æther we say nothing else than that space has to be viewed as a carrier of physical qualities."

– A. Einstein, 1919 in a letter to H. A. Lorentz

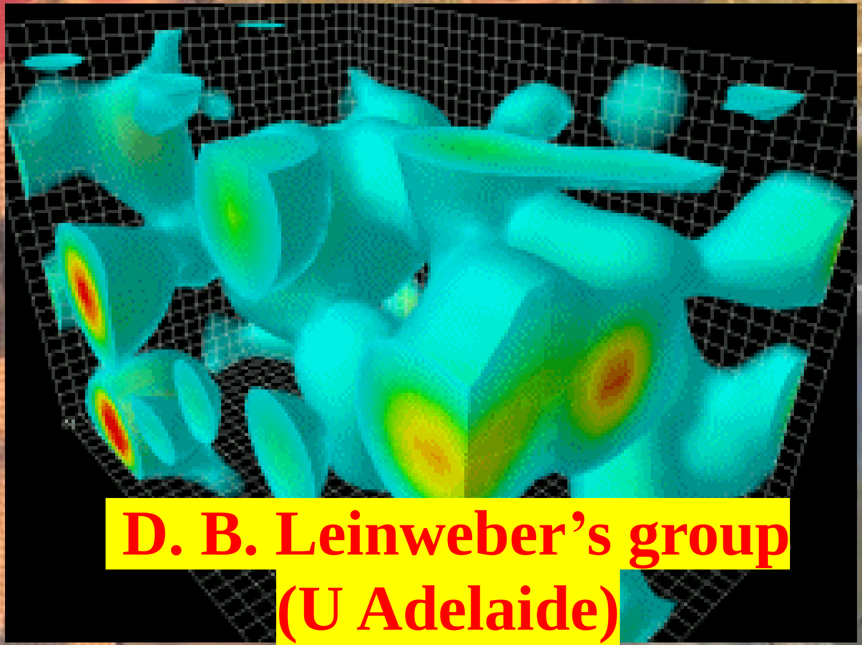
We research:

The modern understanding of the aether as the **"structured quantum vacuum"** in the presence of strong fields.



JOHANN RAFELSKI
BERNDT MÜLLER

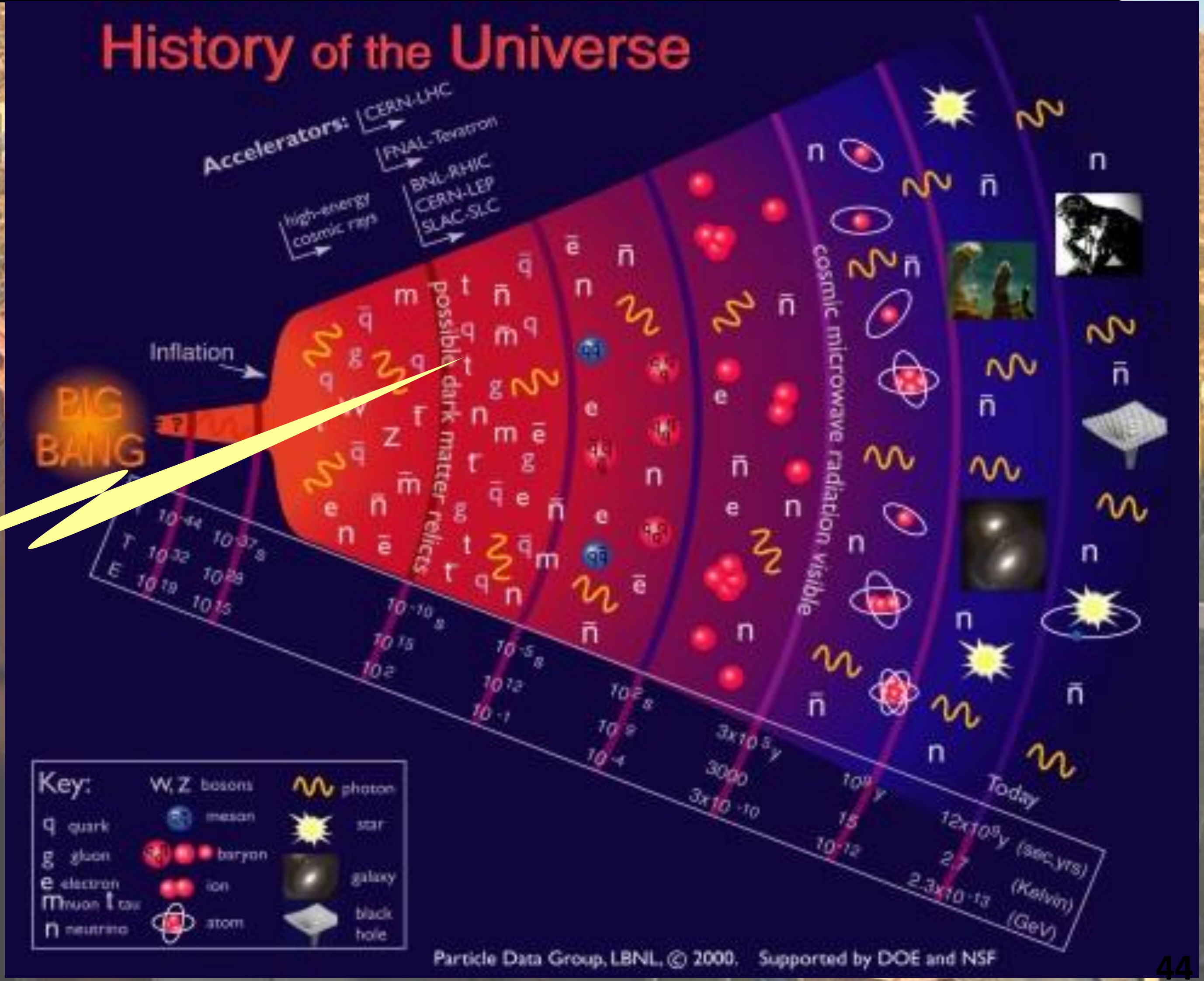
THE STRUCTURED VACUUM
THINKING ABOUT NOTHING



D. B. Leinweber's group (U Adelaide)

How was matter created?
Matter emerges from quark-Gluon plasma

- After the Big-Bang, the “vacuum” was **different** until about 30 μs as expansion cooled the temperature T to a value at which the vacuum changed and our matter “froze out”.
- At that time the density of matter was about $\sim 10^{16}\,\frac{\mathrm{g}}{\mathrm{cm}^3}$ therefore the energy density $\sim 10\,\frac{\mathrm{GeV}}{\mathrm{fm}^3}$ was well above that of the center of neutron stars, that is ~60 times nuclear energy density.
- The temperature was $T \sim 160$ MeV, that is $\sim 2 \times 10^{12}$ K.



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Quark-gluon plasma (QGP) *probed in heavy-ion collisions*

Scattering damping: κ
Medium 4-velocity: u
Distribution function: f_{eq}

$$\frac{m_\pi^2 c^2}{e\hbar} \approx 3.1 \times 10^{14}\ \mathrm{T}$$

The induced EM fields $F^{\mu\nu}_{ind}$ generated by QGP can be modelled using the Vlasov-Boltzmann equation with scattering term.

$$(p\cdot\partial)f(x,p) + qF^{\mu\nu}_{ext}p_\nu\left(\frac{\partial f(x,p)}{\partial p^\mu}\right) = \kappa(p\cdot u)\left(f_{eq}(p)\frac{n(x)}{n_{eq}} - f(x,p)\right)$$

The induced 4-current $J^\mu_{ind}(k)$, in Fourier modes, is then

$$\tilde{J}^\mu_{ind}(k) = 2N_c\int\frac{d^4p}{(2\pi)^4}4\pi\delta_+(p^2-m^2)p^\mu\sum_{u,d,s}q_f\left(\tilde{f}_f(k,p)-\tilde{f}_{\bar{f}}(k,p)\right)$$

The polarization can then be identified.

$$j_i = \sigma_{ij}E_j$$

Relativistic Ohm's Law in QGP

$$\tilde{J}^\mu_{ind}(k) = \Pi^\mu_\nu\tilde{A}^\nu(k)$$

Strong electromagnetic polarization modifies QGP

C. Grayson, M. Formanek, B. Müller, J. Rafelski, "EM Polarization of Quark-Gluon Plasma" In preparation. (2021)

M. Formanek, C. Grayson, J. Rafelski, B. Müller, "Current-Conserving Relativistic Linear Response for Collisional Plasmas" Annals of Physics 434 (2021) doi:10.1016/j.aop.2021.168605 [arXiv:2105.07897]

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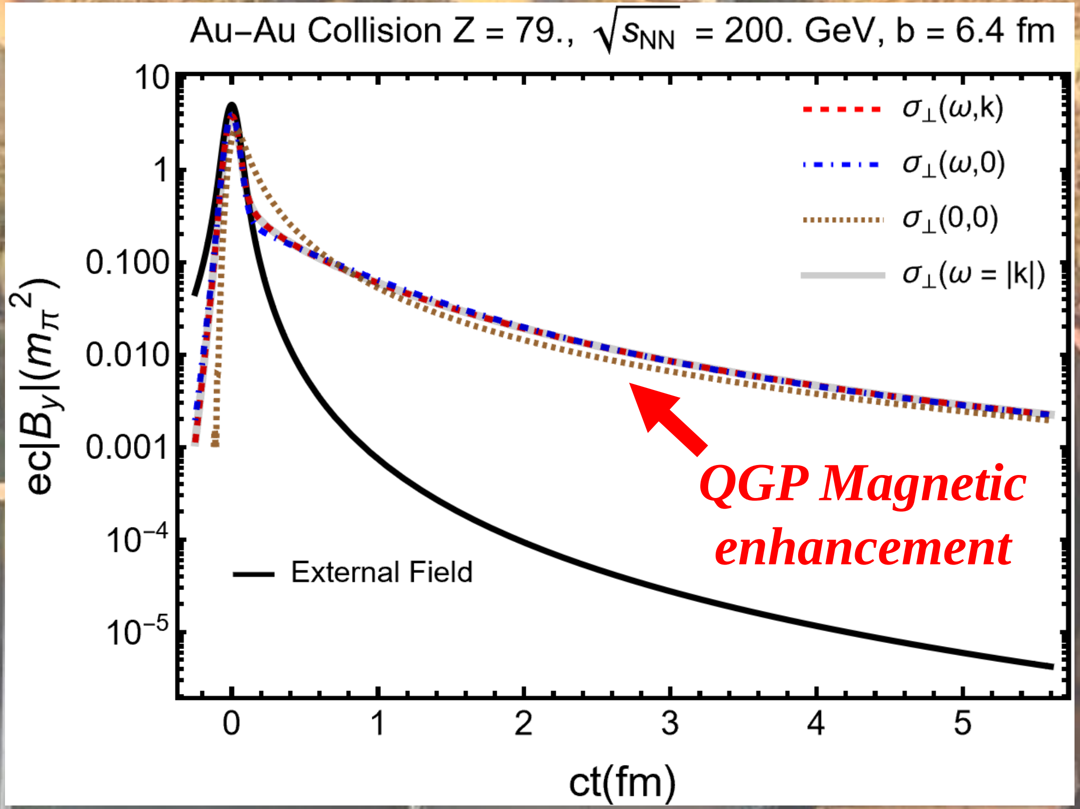
QGP magnetic enhancement: Magnetic field at the geometric origin of particle collision



QGP Magnetic enhancement

The magnetic field spikes and then drops at the origin of the collision as a function of time.

$$\sigma_\perp = \frac{i m_D^2}{4\omega}\left(\frac{\kappa^2}{\omega^2}\xi \ln \xi + \frac{i\kappa}{\omega}(\xi + 1)\right)$$

Transverse Conductivity

$$\xi = 1 - \frac{2i\omega}{\kappa}$$

The relativistic ion field samples the polarization on the light-cone.

C. Grayson, M. Formanek, B. Müller, J. Rafelski, "EM Polarization of Quark-Gluon Plasma" In preparation. (2021)

M. Formanek, C. Grayson, J. Rafelski, B. Müller, "Current-Conserving Relativistic Linear Response for Collisional Plasmas" Annals of Physics 434 (2021) doi:10.1016/j.aop.2021.168605 [arXiv:2105.07897]

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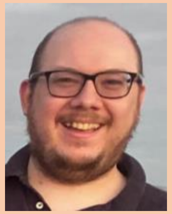
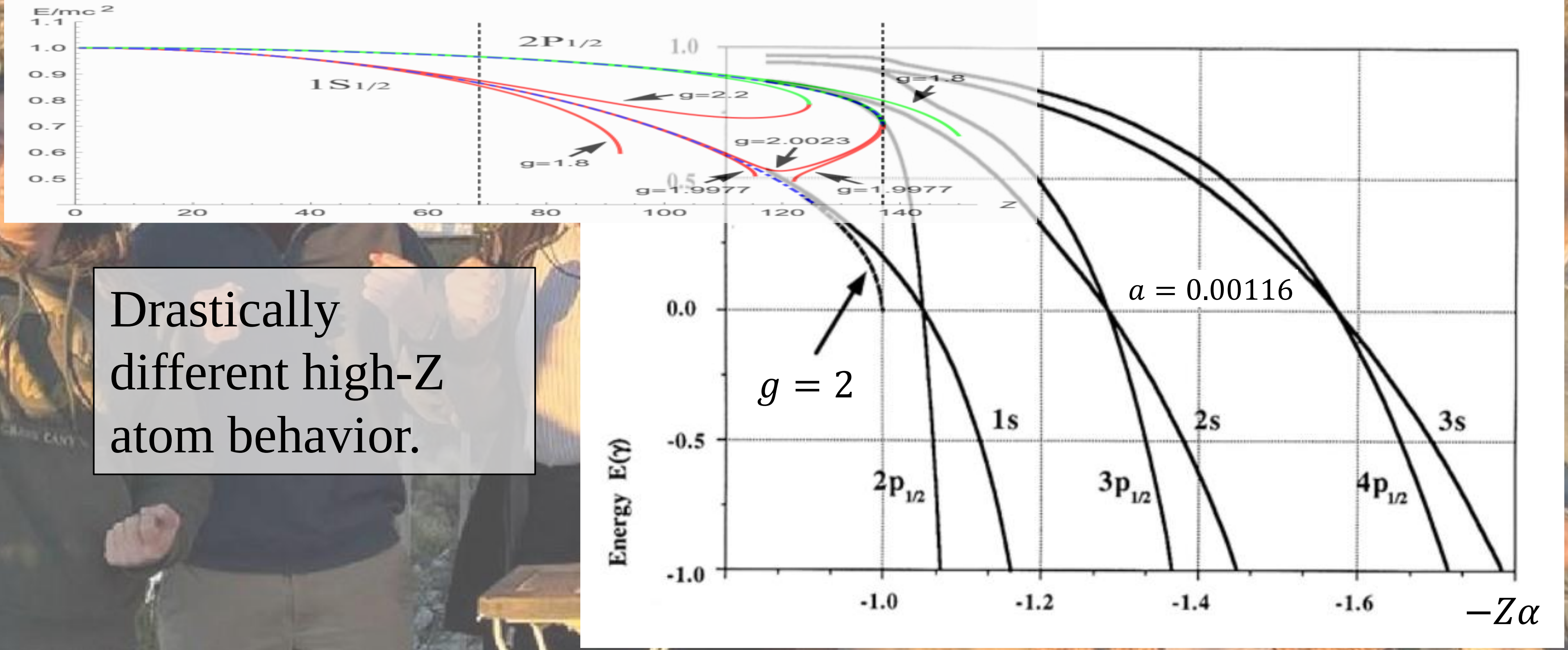


Strong Coulomb field eigen-energies

KGP and DP Spectrum with Same Scaling



Drastically different high-Z atom behavior.

A. Steinmetz, M. Formanek, and J. Rafelski. “Magnetic dipole moment in relativistic quantum mechanics.” EPJA 55.3 (2019): 1-17.

B. Thaller. The Dirac equation. Springer Science & Business Media, 2013.

Magnetic moment and major modification of pair production

KGP introduces corrections into Euler-Heisenberg (**EH**) action:

- Pair production modification due to periodicity of g.

$$L_{EH} = -\frac{1}{8\pi^2}\int_{\delta}^{\infty}\frac{ds}{s^3}\,e^{-m_e^2 s}\left(\frac{abe^2s^2\cos\left[\frac{g}{2}eas\right]\cosh\left[\frac{g}{2}ebs\right]}{\sin[eas]\sinh[ebs]}-1\right)$$

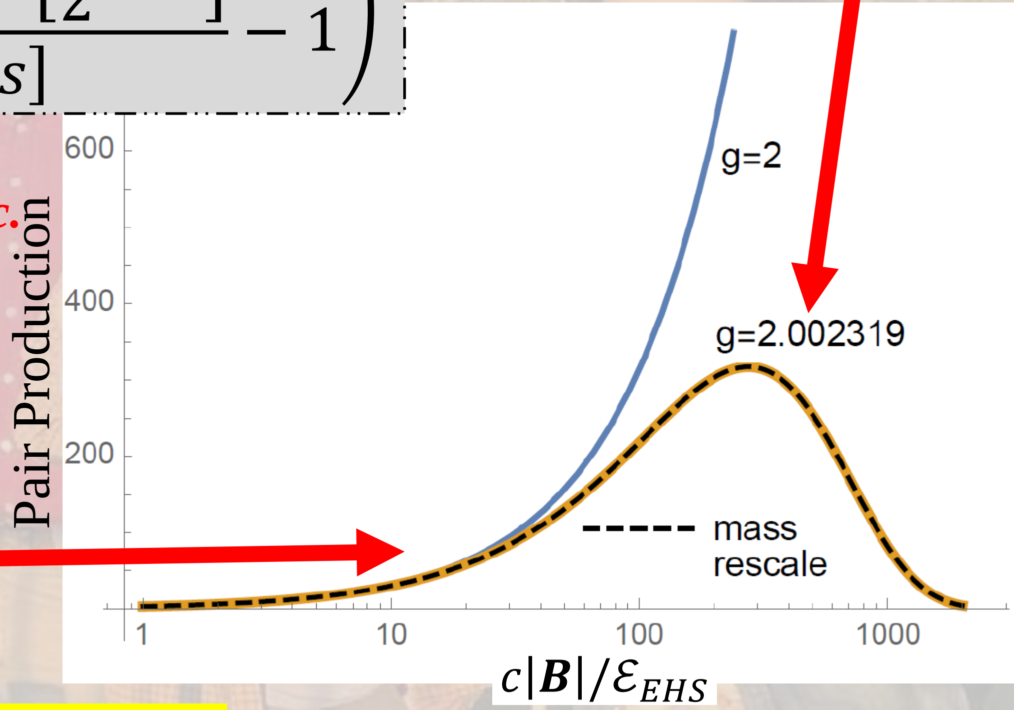
Origin of electron mass? Higgs and electromagnetic.

$$\mathcal{E}_{EHS} = \frac{m_e^2c^3}{e\hbar} = 1.323\times 10^{18}\,\frac{\mathrm{V}}{\mathrm{m}}$$

$$a^2 - b^2 = E^2 - B\hat{}2$$
$$a^2b^2 = (E\cdot B)^2$$

$$m_{rescale}^2c^2 = m_e^2c^2 + \left|\frac{g}{2}-1\right|e\hbar B$$

Modification to pair production



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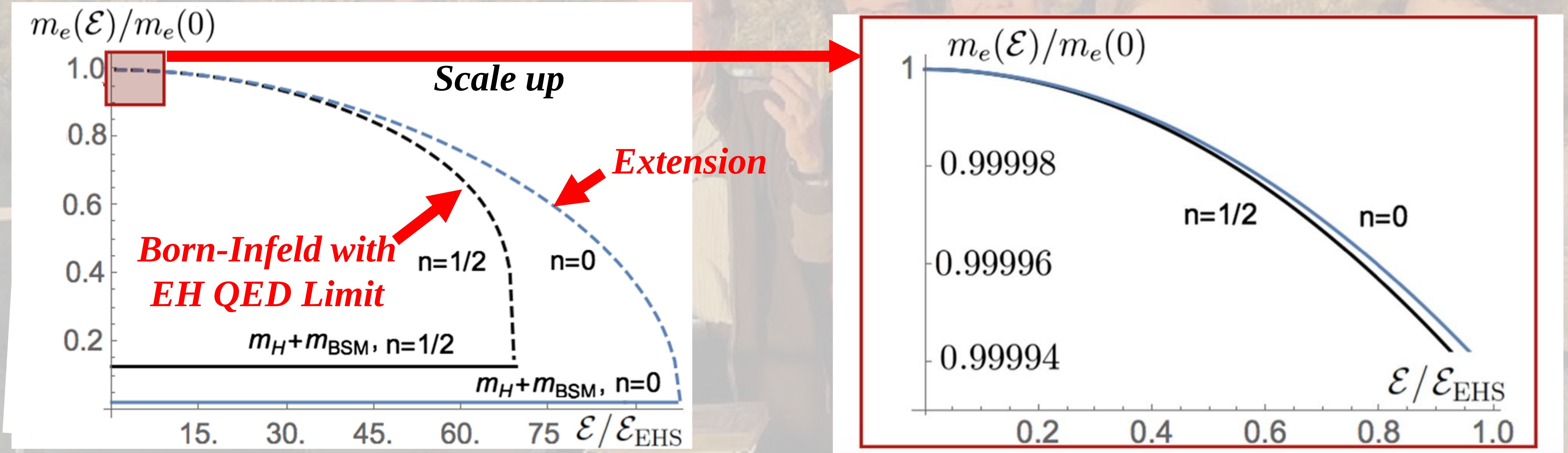
Strong fields as probes of the origin of electron mass

Origin of mass:

- EM and non-EM (Higgs+BSM) mass components
- EM mass melting in external fields
- Self-consistent feedback with nonlinear EM action

Using Born-Infeld model of the electron.



EM mass content controlled by model parameter $\boldsymbol{n}$

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